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IN TWO VOLUMES.

VOL. II.—Containing the *Praxis*.

BY FRANCIS BAILY.

LONDON:

SOLD BY JOHN RICHARDSON, ROYAL EXCHANGE.

1813.

Printed by Richard Taylor and Co., Shoe-Lane, London.

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CHAPTER XII.

Practical Questions to illustrate the Use of some of the preceding Problems.

QUESTION I.

§ 371. To find the *probability* that a life or lives, of any given age, will continue in being to the end of any given term; according to any given table of observations.

SOLUTION.

In the case of a *single* life, this probability is a fraction whose denominator is the number of persons living at the given age; and whose numerator is the number of persons living at an age older by the given term than the given age.

In the case of *joint* lives it is the product of the probabilities that each of the single lives shall continue in being to the end of the given term. See § 23.

Example 1. The probability that a person, whose age is 20, shall attain to the age of 50, or live 30 years, is, according to the observations of *M. De Parcieux* as given in Table III, equal to $\frac{111}{111}$. And the

probability that a person, whose age is 40, shall attain to the age of 70, or live 30 years, is, according to the same observations, equal to $\frac{310}{657}$.

But the probability that *both* those persons shall live to the end of 30 years is equal to $\frac{581}{814}$ multiplied by $\frac{310}{657}$: that is, equal to $\frac{180110}{534792}$.

Example 2. The probability that a man aged 46 shall attain to the age of 56, or live 10 years, is, according to the observations made in *Sweden*, as given in Table XIV, equal to $\frac{3096}{3991}$. And the probability that a woman aged 40 shall attain to the age of 50, or live 10 years, is, according to the same observations, equal to $\frac{4027}{4733}$.

But the probability that *both* those persons shall live 10 years is equal to $\frac{3096}{3991}$ multiplied by $\frac{4027}{4733}$: that is, equal to $\frac{12467592}{18889403}$.

Example 3. The probability that *each* of three lives aged 20, 30, and 40 shall live 15 years, is, according to the observations made at *Northampton*, as given in Table XXV, equal to $\frac{4010}{5132}$, $\frac{3248}{4285}$, and $\frac{3446}{9635}$ respectively.

But the probability that *all* those lives shall continue so long is equal to the product of the three fractions into each other: whence, such probability will be denoted by $\frac{31883927040}{81891385700}$.

SCHOLIUM.

§ 372. Having thus found the probability that any single or joint lives will continue in being to the

end of any given term, we may readily determine the probability that *one or the other* of them will live so long. For, in the case of two lives, the probability here alluded to will be equal to the *difference* between the probability that the joint lives will continue to the end of the term and the sum of the probabilities that each of the single lives will continue so long.

Thus, in the first example, the probability that one or other of two lives, aged 20 and 40, will continue 30 years is equal to $\frac{189110}{534798}$ subtracted from $\frac{634057}{534798}$ (or from the sum of the two quantities $\frac{581}{814}$ and $\frac{310}{657}$):* which leaves $\frac{453947}{534798}$ for the probability required.

And the probability that one or other of the two lives, mentioned in the second example, will continue 10 years, is equal to $\frac{12467594}{18889403}$ subtracted from $\frac{30725195}{18889403}$ (or from the sum of the two quantities $\frac{3096}{3991}$ and $\frac{4937}{4733}$): which leaves $\frac{18257533}{18889403}$ for the probability required.

In like manner the probability that some one or other, out of three given lives, will continue to the

* These fractions, reduced to a common denominator, are $\frac{381717}{534798}$ and $\frac{252340}{534798}$, the sum of which is equal to $\frac{634057}{534798}$. But it is tedious to operate in this way, and I have adopted it in the present instance for the sake of illustration only. The best method of finding the probabilities, both for single and joint lives, is by means of logarithms; and I would here observe that the logarithm of the denominator subtracted from the logarithm of the numerator will give the logarithm of the probability required: which logarithm will always have a *negative index*.

end of any given term, is found by subtracting the sum of the probabilities that each pair of joint lives will continue so long, from the sum of the probabilities that each single life and that the three joint lives will continue the given time: agreeably to the principles laid down in Prob. II.*

QUESTION II.

§ 373. To find the *expectation* of any given life (or lives) receiving a given sum, at the end of any given term.

SOLUTION.

Multiply the present value of the given sum by the probability that the given life (or lives) will continue to the end of the given term: the product will be the answer required. See the note in page 44.

Example 1. What is the present value of £1 to be received at the end of 30 years, provided a person, now aged 20, be then alive: interest being reckoned at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

* I shall here mention, by way of note, that the probability that any *two out of three* given lives will continue to the end of any given term, is equal to twice the probability that the three joint lives shall continue the given time, subtracted from the sum of the probabilities that each pair of joint lives shall continue the same period: agreeably to what has been said in the investigation of Prob. III.

The present value of £1 to be received at the end of 30 years, without any contingency, is by Table LVIII equal to ·26700; and the probability that a person aged 20 will live 30 years is, by the preceding Question, equal to $\frac{5}{8}\frac{6}{1}\frac{1}{4}$: therefore, these two quantities multiplied together* will produce ·1906 for the value required.

In like manner the expectation of receiving that sum at the end of the same period, provided a person, aged 40, lives so long, is equal to $\frac{3}{6}\frac{1}{3}\frac{9}{7}$ multiplied by ·26700; which produces ·12598 for the value in this case required.

But, if the expectation depended on *both* those lives continuing to the end of the term; then $\frac{1}{5}\frac{8}{3}\frac{9}{4}\frac{1}{7}\frac{1}{9}\frac{9}{8}$, multiplied by ·26700, will produce ·08992 for the value required.

And, if it had depended on *either* of those lives continuing to the end of the term, then $\frac{4}{5}\frac{5}{3}\frac{3}{4}\frac{9}{7}\frac{4}{9}\frac{7}{8}$ (or the value found by the Scholium in page 357) being multiplied by ·26700 will produce ·22663 for the value of the expectation in such case required.

Example 2. A man aged 46 will at the expiration of a lease, which has 10 years to run, be entitled to a fine of £1,† provided he be then alive: what is his

* The method of multiplying a vulgar fraction by a decimal fraction is, to multiply the *decimal* by the *numerator* of the vulgar fraction, and to divide the product by the *denominator* of the same.

† I have taken the fine equal to *one* pound, because the quantities which result from this assumption will be often referred to in the course of the present chapter: but, it is easy to see that the answer here obtained, being multiplied by *any other* fine,

expectation of receiving the same: interest being reckoned at 4 per cent, and the probabilities of living as observed in *Sweden?*

The present value of £1 certain to be received 10 years hence is, by Table LVIII, equal to ·67556; and the probability that a man aged 46 will live 10 years is, by the preceding question, equal to $\frac{3086}{5091}$: therefore these two quantities multiplied together will produce ·52406 for the value required.

Had the fine depended on the life of his wife aged 40, then $\frac{4097}{4733}$ multiplied by ·67556, will produce ·57479 for the value in this case required.

But had it depended on their *joint* lives continuing to the end of the given term, then $\frac{12467503}{18860403}$ multiplied by ·67556, will produce ·44589 for the value in such case required.

And had it depended on *either* of those lives continuing so long, then $\frac{18257533}{18860403}$ multiplied by ·67556 will produce ·65296 for the value in this case required.

SCHOLIUM.

§ 374. By means of the general solution here given may be determined all questions relative to the value of such sums as ought to be given for the

would give the present value of such other fine. Thus, if the fine were £100, the present value of the same, if depending on the life of the man, would be equal to 52·406 or £52 : 8 : 1; and, if depending on the life of the woman, would be equal to 57·479 or £57 : 9 : 7.

Endowments of Children. Thus, suppose a person has a son aged 11, for whom he wishes to secure £100 on his coming of age; the sum which he ought to pay down for the assurance of the same (reckoning interest at 5 per cent, and the probabilities of living as according to *M. De Parcieux*) is equal to $\frac{806}{872}$ multiplied by 61·391; which produces 56·744, or £56 : 14 : 10 for the answer required.

In the table of rates published by the *Globe Assurance* Company, and by the *Provident Institution**, the sums demanded for the Endowments of Children are in general full as much as (and from the age of 9 years and upwards are even *more* than) the present values, at 5 per cent, of £100 *certain* to be received at the end of the given term *without any contingency.* For instance, £62 : 11 : 2 is required in ready money by the *Globe*, and £62 by the *Provident*, for the payment of £100 on the event of a child, aged 11, arriving at the age of 21 years: whereas either of these sums put out to interest at 5 per cent would amount to *more* than £100 at the end of that period, without the liability to loss in case the child should happen to die before that time!!! No person can, I think, be so blind to his own interest as to risk his money in this absurd way.

* The other Companies have not published *their* rates for the Endowments of Children: but, from the similarity of most of the rates at the various Offices, we may conclude that there is no great difference on this point.

QUESTION III.

§ 375. To find the value* of an annuity on any *Single* life.

SOLUTION.

This value is determined by inspection: for, in either of the Tables which show the values of annuities on any single life, we shall find the value required set down against the age of the given life, according to the several rates of interest at the top of each column.

Example 1. The value of an annuity on a life aged 20, reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux*, is, by Table V, equal to 16·624; or about $16\frac{5}{8}$ years purchase.†

* By the *value* of an annuity I mean the *number of years purchase* that such annuity is worth: agreeably to what I have already observed in the note in page 27: and as this mode of expression is used in all the subsequent questions, it will be necessary to bear this observation in mind.

† The *number of years purchase* being multiplied by the annuity will give the total present value of the same. Thus, if the annuity in the present instance were £50 per annum: then 16·624 multiplied by 50 would give 831·200, or £831 : 4 ; 0 for the value of the same. Or, if the annuity had been £4 : 10 : 0 per annum; then 16·624 multiplied by 4·5 would give 74·808 or £74 : 16 : 2 for the value in this case required. This method is universal, and applies to all cases of annuities, whether present or in reversion; whether temporary or deferred: and therefore it will be sufficient, in all the subsequent examples, to deduce the value of an annuity

Had the life been 40 years of age, the value would have been equal to 14·254. Or had the rate of interest been in each case 5 per cent, the values would have been equal to 15·469 and 13·459 respectively.

Example 2. The value of an annuity on the life of a *man* aged 46, reckoning interest at 4 per cent and the probabilities of living as observed in *Sweden* is, by Table XVI, equal to 12·297; or rather more than 12¼ years purchase.

Had the annuity been on the life of a *woman* aged 40, the value would have been equal to 14·401. Or had the rate of interest in each case been 5 per cent, the values would have been 11·153 and 12·856 respectively.

QUESTION IV.

§ 376. To find the value of an annuity on *two Joint lives*.

of *one pound* per annum: or, in other words, to find the *number of years purchase*.

Having thus found the number of years purchase that ought to be given for an annuity, we may readily determine the *annuity* that ought to be given for any given *sum* invested, merely by dividing such sum by the number of years purchase. Thus, if a person wished to lay out £4000 in the purchase of such an annuity as the one mentioned in the text, the annuity which he ought to receive for that money will be found by dividing 4000 by 16·624: whence 240·616, or £240 : 12 : 4 will be the annuity required. This method is likewise universal, and therefore it will be unnecessary to repeat it in any of the subsequent cases. The same principles will apply to the value of reversionary *sums*; for which, see Question XXVII.

SOLUTION.

Look in the Tables which show the values of annuities on two joint lives of all ages; and if the two lives have the same common age, or if their difference of age comes within the limits of those tables, the value of an annuity on their joint continuance will be found expressed therein.

Example 1. The value of an annuity on two joint lives aged 20 and 40, interest being reckoned at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux*, is, by Table IX, equal to 12·545; or rather more than $12\frac{1}{2}$ years purchase.

Had both the lives been 20 years of age, the value would have been, by Table VI, equal to 14·004: or, had they been both 40 years of age, the value would have been 11·710.

Example 2. The value of an annuity on the joint lives of a man aged 46 and his wife aged 40, reckoning interest at 4 per cent and the probabilities of living as observed in *Sweden*, is, by Table XVIII, equal to 10·286.

Had both the lives been 40 years old the value would have been, by Table XVII, equal to 10·964: or, had they been both 46 years old, the value would have been 9·736.*

* The values of annuities on the joint lives in Tables XVII to XXIV are deduced from the probabilities of living amongst males and females *collectively:* and therefore do not show the *true* values of annuities on two joint lives, one of which is a *male* and the other a *female*. Tables formed upon this latter principle are still a *desideratum*. See the example in page 33.

SCHOLIUM.

§ 377. If the *difference* of age between the two lives is any number of years not given in the tables, the required value may be easily obtained by means of the following rule.

Find, by the tables, the value of an annuity on two joint lives whose difference of age is *greater* than, but at the same time *nearest* to, the difference of age between the proposed lives; and the oldest of which is of the same age with the oldest of the proposed lives. Find also, by the same tables, the value of an annuity on two joint lives whose difference of age is the *next less* to that just mentioned; and the oldest of which is, in like manner, of the same age with the oldest of the proposed lives. Then will the 1st, 2nd, 3rd, &c, arithmetical *mean** between the least and the greatest of these two values be the value required; according

* The tables for the values of annuities on two joint lives, according to the *Northampton* observations, are the only ones where the difference of age is so small as *five* years. In the tables deduced from the observations in *Sweden*, the difference of age is *six* years: and in those deduced from the observations of *M. De Parcieux*, the difference of age is *ten* years. Consequently the 1st, 2nd, 3rd, &c arithmetical mean between the least and greatest of any two values, according to the *Northampton* tables, will be equal to the least value increased by 1, 2, 3, &c *fifths* of their difference: but according to the *Swedish* tables, it will be equal to the least value increased by 1, 2, 3, &c *sixths* of their difference: and according to the tables of *M. De Parcieux*, it will be equal to the least value increased by 1, 2, 3, &c *tenths* of their difference.

as one of the proposed lives is 1, 2, 3, &c years younger than the other.

Example 1. Let it be required to find the value of an annuity on two joint lives aged 32 and 50; at the rate of $4\frac{1}{2}$ per cent interest, and according to the probabilities of life as observed by *M. De Parcieux?*

That difference of age which is *greater* than the difference between these lives, but at the same time *nearest* to it, is 20: and the value of an annuity on two joint lives whose difference of age is 20 years, and the oldest of which is of the same age with the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 30 and 50) is, by Table IX, equal to 10·611. And the value of an annuity on two joint lives whose difference of age is next less to 20 (that is, whose difference of age is 10 years) and the oldest of which is of the same age with the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 40 and 50) is, by Table VIII, equal to 10·274. Therefore, these being the values of an annuity on two joint lives aged 30 and 50, and on two joint lives aged 40 and 50; it is evident that the value of an annuity on two joint lives, aged 32 and 50, will be nearly equal to the least of these two values increased by 8 *tenths* of the difference between them: or (which is the same thing) equal to the greatest value diminished by 2 tenths of their difference. Now, the difference between these values is equal to ·337; *one* tenth of which is equal to ·0337, and *two* tenths are therefore equal to ·067.

Consequently 10·611, diminished by ·067, will leave 10·544 for the value required of an annuity on the two joint lives aged 32 and 50.

Example 2. Let it be required to find the value of an annuity on two joint lives aged 20 and 60; at the rate of 4 per cent interest, and according to the probabilities of living as observed in *Sweden?*

The difference of age which is greater than the difference between these lives, but at the same time nearest to it, is 42: and the value of an annuity on two joint lives whose difference of age is 42 years, and the oldest of which is of the same age with the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 18 and 60) is, by Table XXIV, equal to 8·208. And the value of an annuity on two joint lives whose difference of age is 6 years less than 40, and the oldest of which is likewise of the same age with the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 24 and 60) is, by Table XXIII, equal to 8·097. Therefore, these being the values of an annuity on two joint lives aged 18 and 60, and on two joint lives aged 24 and 60; it follows that the value of an annuity on the two joint lives 20 and 60 will be nearly equal to the least of these two values increased by 4 *sixths* of the difference between them. Now, their difference being equal to ·111, it follows that *one* sixth of such difference will be ·0185; and *four* sixths of such difference will be ·074: which being added to 8·097 will give 8·171 for the required value of an annuity on the two joint lives aged 20 and 60.

Example 3. What is the value of an annuity on two joint lives aged 26 and 60; reckoning interest at 5 per cent and the probabilities of life as observed at *Northampton?*

The difference of age which is greater than the difference between these two lives, but at the same time nearest to it, is 35: and the value of an annuity on two joint lives whose difference of age is 35, and the oldest of which is equal to the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 25 and 60) is by Table XXXV equal to 7·383. And the value of an annuity on two joint lives whose difference of age is 5 years less than 35, and the oldest of which is also of the same age with the oldest of the proposed lives (that is, the value of an annuity on two joint lives aged 30 and 60) is, by Table XXXIV, equal to 7·292. Therefore, these being the values of an annuity on two joint lives aged 25 and 60, and on two joint lives aged 30 and 60; it follows that the value of an annuity on the two joint lives 26 and 60 will be nearly equal to the least of these values increased by 4 *fifths* of the difference between them: or nearly equal to the greatest of these values decreased by 1 *fifth* of their difference. Now, this difference being ·091, it is evident that *one* fifth of it is equal to ·018; which being deducted from 7·383,* will give 7·365 for the value required of an annuity on the two joint lives 26 and 60.

§ 378. Since the tables of the values of annuities

* Or we may add four fifths to the least value, which would give the same result.

on two joint lives according to the observations of *M. De Parcieux* are calculated only for such lives whose difference of age is *ten* years, it is evident that the method just laid down (for determining the values of annuities on two joint lives, whose difference of age is any intermediate number) will not be quite so correct as from those tables, calculated according to the observations of life in *Sweden*, where the difference of age is *six* years. Neither will these latter ones show the value, for such intermediate ages, so correctly as the tables calculated according to the observations of life at *Northampton*, where the difference of age is *five* years. In neither case will the error be very considerable: but in the latter case particularly (where the tables show the values of annuities on two joint lives of all ages whose difference is not more than 5 years) the error is so trifling as to be not worth considering. This will evidently appear from the following comparison (given by Dr. Price in his *Observations on Reversionary Payments*, vol. 2, page 359,) of the values of annuities on two joint lives of the ages therein mentioned, deduced from the *Northampton* observations, interest at 3 per cent.

Ages.	Value by Rule.	Correct Value.
18—14	14·972	14·978
18—15	14·858	14·864
18—16	14·744	14·744
18—17	14·630	14·626
45—31	10·862	10·869
45—32	10·802	10·811

Ages.	Value by Rule.	Correct Value.
45—33	10·742	10·751
45—34	10·682	10·688
66—27	7·092	7·095
66—28	7·076	7·080
66—29	7·060	7·063
66—30	7·044	7·046

In the higher rates of interest the agreement is greater.

Dr. Price was enabled to make this comparison by the Tables in the office of the *Equitable Society*; where, in order to lay the foundation of accuracy in conducting the business of the office, it has been thought necessary to compute minutely to four places of decimals the values by the *Northampton* observations, at 3 per cent, of two joint lives *for every possible difference of age.*

§ 379. When one of the given lives is under 10 years of age, we ought, in deducing the values agreeably to this rule, to attend particularly to the order of the difference between the values taken from the Tables: that is, to observe whether such difference is *in*creasing or *de*creasing. For instance, suppose it is required to determine the value of an annuity on two joint lives aged 9 and 30, interest at 3 per cent, and the probabilities of living as at *Northampton:* the rule directs us to find the value of an annuity on two joint lives aged 5 and 30, and on two joint lives 10 and 30, which are respectively equal to 13·762 and 14·150;

and that ·078 (or one fifth of the difference between them) being subtracted from the latter value, will give 14·072 for the value of an annuity on two joint lives aged 9 and 30. But the following comparison will show this to be incorrect: for if we take out the values of annuities on the several joint lives as under, viz.

5—30 = 13·762
10—30 = 14·150
15—30 = 13·734
20—30 = 13·286
25—30 = 12·966
30—30 = 12·589

it will be seen that (beginning at the bottom) the values gradually *in*crease till we come to the age of 10 and 30; and therefore that the value of an annuity on any two joint lives, one of which is 30 years of age and the other of any age between 10 and 30, will be deduced accurately enough by means of the rule above given. And this also would be the case with respect to the value of annuities on any two joint lives, one of which is 30 years and the other of any age *below* 10 years of age, provided the *de*crease commenced exactly at the age of 10 years: but it is probable that the decrease does not begin to take place till about the joint ages of 8 and 30;* and consequently that the value of an annuity on two joint lives aged 9 and 30 is *greater* than 14·150, instead of being *less*. The

* The period, at which this decrease commences, varies according to the rate of interest and according to the difference between the ages of the two lives.

proper method therefore of finding the value of an annuity on the two joint lives aged 9 and 30 will be to take ·083 (or one fifth of the difference between 14·150 and 13·734) and *add* it to 14·150; which will give 14·233 for the value of an annuity on the two joint lives aged 9 and 30. These cases have never yet been noticed by any preceding writer; although they frequently occur in practice.

QUESTION V.

§ 380. To find the value of an annuity on *three Joint* lives.

SOLUTION.

Look in Tables XLIII and XLIV; and if the three lives have the same common age, or if their difference of ages be 10 and 20 years, the value of an annuity on their joint continuance will be found expressed therein.

Example. The value of an annuity on three joint lives aged 20, 30 and 40, reckoning interest at 4 per cent and the probabilities of living as at *Northampton*, is equal to 8·986: but had all the lives been 20 years of age, the value would have been equal to 10·342; or had they all been 40 years of age, the value would have been equal to 7·865.

SCHOLIUM.

§ 381. It unfortunately happens that the two tables above mentioned are the only ones, that have been published, for determining the values of annuities on three joint lives. The labour of computing such tables is so very great, and the combinations of ages are so various, that it will probably be a long time before any person will undertake to finish what has been here begun: and till that is the case we may make use of the following general and very easy rule, given by Mr. Simpson, for finding the values of annuities on any *three*, from the values on any *two*, joint lives.

" Let A be the youngest, and C the oldest of the " three proposed lives. Take the value of an annuity " on the two joint lives B and C, and find the age of " a single life D of the same value. Then find the " value of an annuity on the two joint lives A and D, " which will be the value required."

Example. What is the value of an annuity on three joint lives aged 10, 20, and 30; interest at 4 per cent, and the probabilities of living as at *Northampton?**

The value of an annuity on the two joint lives aged

* It will readily appear that we can obtain the values of annuities on *three* joint lives more correctly from the *Northampton* tables of *two* joint lives, than from any other observations; because they are, as yet, the most comprehensive, and include the greatest variety of combined ages.

20 and 30 is, by Table XXX, equal to 11·873; which, being compared with the values in Table XXVII, will be found equal to the value of an annuity on a single life D aged $47\frac{17}{205}$,* or 47 years and 1 month. And the value of an annuity on the joint lives A and D (that is, on two joint lives aged 10 and $47\frac{17}{205}$) is, by the rule in the preceding scholium, equal to 10·474;† which is the value required.

Had the two oldest lives been both 40 years of age, and the youngest 20, the value of an annuity on the joint lives of the two former would, by Table XXVIII, be equal to 9·820; answering to a single life D aged $56\frac{157}{229}$. And the value of an annuity on the joint lives A and D (that is, on two joint lives aged 20 and $56\frac{157}{229}$) is, by the rule alluded to in the last note,

* The value, in Table XXVII, which is next greater than 11·873, is 11·890; which is the value of an annuity on a single life aged 47. The difference between these values, or 17, is the *numerator* of the fraction: and the *denominator* is the difference between 11·685 (or the next less value to 11·873) and 11·890.

† The value of an annuity on two joint lives aged 10 and 47 is, by the rule in the preceding scholium, equal to 10·485: and the value of an annuity on two joint lives aged 10 and 48 is by the same rule equal to 10·356. The difference between these two values, or ·129, being multiplied by $\frac{17}{205}$ will give ·011; which being subtracted from 10·485 will leave 10·474 for the value required. This shows the true method of proceeding in such cases; but, if this fraction be either very small or does not differ much from unity, the error will not be considerable if (for the sake of more expedition) D is always taken for that age, whether greater or less, which answers most nearly to the value of the annuity on the joint lives B and C, without regarding the fraction.

equal to 8·601: which is the value required of an annuity on three joint lives aged 20, 40, and 40.

Or, had the two youngest lives been 20, and the eldest 40 years of age; then the value of an annuity on two joint lives aged 20 and 40 would, by Table XXXII, be equal to 10·924: answering to a single life D aged $51\frac{133}{208}$. And the value of an annuity on the two joint lives A and D (that is, on two joint lives aged 20 and $51\frac{133}{208}$) is, by the rule alluded to in the preceding note, equal to 9·406: which is the value required of an annuity on three joint lives aged 20, 20 and 40.

The following table (computed from the probabilities of life as observed at *Northampton*, and reckoning interest at 4 per cent) will show how nearly the rule, above explained, approximates to the true values as given in Tables XLIII and XLIV.

Ages.	Value by Rule.	Correct Value.
10—20—30	10·474	10·438
15—25—35	9·836	9·738
20—30—40	9·097	8·986
25—35—45	8·390	8·313
30—40—50	7·651	7·571
35—45—55	6·884	6·816
40—50—60	6·046	5·994
45—55—65	5·175	5·145
50—60—70	4·235	4·219
55—65—75	3·308	3·298
10—10—10	12·206	12·200
15—15—15	11·376	11·274
20—20—20	10·516	10·342

Ages.	Value by Rule.	Correct value.
25—25—25	9·937	9·796
30—30—30	9·351	9·221
35—35—35	8·703	8·585
40—40—40	7·983	7·865
45—45—45	7·243	7·126
50—50—50	6·433	6·317
55—55—55	5·637	5·550
60—60—60	4·817	4·755
65—65—65	3·936	3·914
70—70—70	3·010	2·995
75—75—75	2·118	2·119

From which it may be inferred that this rule will give the values of annuities on three joint lives generally within a *ninth* or a *tenth*, and sometimes within less than a *twentieth* part of a year's purchase. It may also be observed that when the oldest of the three ages does not exceed 75, and the youngest is not less than 10, the error falls on the side of *excess:* and, consequently, that if ·05 (or a twentieth part of a year's purchase) be deducted from the values by the rule, we shall obtain the true value, in some cases, almost exactly; and, in most cases, much more nearly.

QUESTION VI.

§ 382. To find the value of a *Deferred* annuity on any *single* or *joint* lives.*

* This Question is of considerable utility in enabling us to determine the best means of providing Annuities for the *benefit of Old Age;* as will be more fully explained in the following Chapter.

SOLUTION.

Find the value of an annuity on a life, or joint lives, as many years older than the given life, or joint lives, as are equal to the term during which the annuity is deferred; find also the expectation of the given life, or joint lives, receiving £1 at the end of that term: the product of these two quantities will be the answer required. See § 45.

Example 1. A person aged 20 wants to purchase an annuity for what may happen to remain of his life after the term of 30 years: what is the present value of the same, reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of an annuity on a life aged 50, is, by Table V, equal to 11·921; and the expectation of a life, aged 20, receiving £1 at the end of 30 years, is, by Question II, equal to ·1906: therefore 11·921 multiplied by ·1906 will produce 2·272 for the number of years purchase required.

Had the life been 40 years of age, the value would have been equal to 6·221 multiplied by ·1260; which would produce ·784 for the value required.

Example 2. A man now aged 46 will at the end of 10 years come into possession of an annuity on his own life: what is the present value of the same, reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on a male aged 56 is, by Table XVI, equal to 9·717; and the expectation that a man aged 46 will receive £1 at the end of 10 years is, by Question II, equal to ·5241: therefore these two quantities being multiplied together will give 5·093 for the value required.

Had the annuitant been a female aged 40, then 12·049 multiplied by ·5748 would give 6·926 for the value in this case required.

Example 3. Two persons aged 20 and 40 wish to purchase an annuity for the remainder of their joint lives after 30 years: what ought they to give for the same, reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux*?

The value of an annuity on two joint lives aged 50 and 70 is, by Table IX, equal to 5·517; and the expectation of two joint lives, aged 20 and 40, receiving £1 at the end of 30 years is, by Question II, equal to ·0899: therefore the product of these two quantities will give ·496 for the value required.

Example 4. A man aged 46 together with his wife aged 40 are entitled to an annuity on their joint lives, to commence at the end of 10 years: what is the value of their interest therein, taking the probabilities of life as observed in *Sweden*, and the rate of interest at 4 per cent.?

The value of an annuity on the joint lives of two persons, a man aged 56 and a woman aged 50, is, by Table XVIII, equal to 7·874; which being multiplied by ·4459 (or the value of the expectation of

two joint lives, aged 46 and 40, receiving £1 at the end of 10 years as found by Question II) will produce 3·611 for the value required.

SCHOLIUM.

§ 383. If, instead of determining the value of a deferred annuity in a *single* payment, we wish to determine the value of the same in *annual* payments during the term for which the annuity is deferred;* the amount of those annual payments is readily obtained by means of the following rule.

Divide the value of the annuity in a *single* payment, by unity added to the value of a similar temporary annuity for one year less than the given term: the quotient will be the annual payment required. See § 366.

Example 1. A person aged 20 wants to purchase an annuity for what may happen to remain of his life after the term of 30 years: what sum ought he to give *annually* to the end of that term† in order to have the same assured to him; reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

* The first of those annual payments to be made *immediately*, and the remaining ones at the *beginning* of every subsequent year: since this is the usual method of making such annual payments.

† Such annual payments, however, subject to failure, in case the given life becomes extinct before the end of that term.

The value of this deferred annuity in a *single* payment is, by the first example to the Question, equal to 2·272; and the value of a similar temporary annuity for 29 years is, by the rule in the following Question,* equal to 14·161: therefore 2·272 divided by 15·161 will give ·150 for the value of the annual payments during the term deferred.

In like manner we might determine the value in annual payments, of an annuity on the life of a woman for what may happen to remain of it after 10 years; reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden.*

For, the value of this deferred annuity in a *single* payment is, by the first example to the Question, equal to 6·926; and the value of a similar temporary annuity for 9 years is, by the rule just alluded to, equal to 6·900: therefore 6·926 divided by 7·900 will give ·877 for the value of the annual payments required.

Example 2. A man aged 46 and his wife aged 40 are entitled to an annuity on their joint lives, to commence at the end of 10 years, but are willing to surrender their interest in the same for an equivalent annuity (commencing immediately) during such term:

* A more convenient method however of determining *such* temporary annuities is expressed by the following rule. To the value of the deferred annuity add the expectation that the given life or lives shall receive £1 at the end of the given term; subtract the sum from the value of an annuity on the given life or lives: the difference will be the value of the temporary annuity for *one year less than the given term.* See the note in page 350.

what ought that equivalent annuity to be, reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of the deferred annuity on the joint lives is, by the fourth example to the Question, equal to 3·511; and the value of a similar temporary annuity for 9 years is, by the following Question (or the rule in the preceding note) equal to 6·329; therefore 3·511 divided by 7·329, will give ·479 for the value of the annual payments during the term deferred.

QUESTION VII.

§ 384. To find the value of a *Temporary* annuity on any *single* or *joint* lives.*

SOLUTION.

From the value of an annuity on the given single or joint lives, deduct the value of an annuity on the same lives *deferred* during the given term: the remainder will be the value required. See § 47.

Example 1. A person aged 20 buys an annuity for 30 years, on condition that if he dies before the expiration of that term, the annuity shall cease: what ought he to give for the same, reckoning interest at

* I call a *temporary* annuity, one that is to continue during a given *term* only: which term is *less* than that to which it is possible the life or lives may extend. See the note in page 45.

$4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on a life aged 20 is, by Table V, equal to 16·624; and the value of an annuity on the same life, deferred for 30 years, is, by Question VI, equal to 2·272: consequently, this value, subtracted from the former, will leave 14·352 for the answer required.

Had the life been 40 years of age, then ·784 (or the value of an annuity on such life deferred for 30 years, as found by Question VI) deducted from 14·254, would leave 13·470 for the value in this case required.

Or, had these two persons (aged 20 and 40) purchased the annuity on their *joint* lives, then ·496 (or the value of an annuity on such joint lives deferred for 30 years, as found by Question VI) being deducted from 12·545, will leave 12·049 for the value in this case required.

Example 2. A man aged 46 is entitled to the rent of an estate for 10 years, provided he lives so long: what is the value of his interest therein, reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on such life is, by Table XVI, equal to 12·297; and the value of an annuity on the same life, deferred for 10 years, is, by Question VI, equal to 5·093: consequently the difference between these two values, or 7·204, will be the value required.

Had the estate depended on the life of his wife aged

40; then 7·475 (or the difference between 14·401 and 6·926) would be the value of the temporary annuity in this case required.

Or had the estate depended on their *joint* lives; then 6·775 (or the difference between 10·286 and 3·511) would be the value of the temporary annuity in this case required.

QUESTION VIII.

§ 385. To find the value of an annuity on the *Longest* of *two* lives.

SOLUTION.

From the sum of the values of an annuity on the two single lives, subtract the value of an annuity on the two joint lives: the difference will be the value required. See § 56.

Example 1. What is the value of an annuity on the longest of two lives aged 20 and 40; interest at 4½ per cent and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on the two single lives is, by Table V, equal to 16·624 and 14·254 respectively, the sum of which is 30·878; therefore, if from this we subtract 12·545, or the value of an annuity on the two joint lives as found by Table IX, the difference, or 18·333, will be the value required.

Had the ages of the given lives been 50 and 70, the sum of the values of an annuity on their single lives would, by Table V, be equal to 18·142 (that is, equal to 11·921 added to 6·221); and the value of an annuity on their joint lives would, by Table IX, be equal to 5·517: consequently 12·625, or the difference between these two values, would be the value of an annuity on the longest of their two lives.

Had both the lives been 20 years of age, the value of an annuity on their single lives would (according to the same rate of interest &c) have been equal to twice 16·624; that is, equal to 33·248: and the value of an annuity on their joint lives would, by Table VI, be equal to 14·004: therefore the difference between these two values, or 19·244, would be the number of years purchase in this case required.

Example 2. What is the value of an annuity on the longest of two lives, a man and his wife, the former aged 46 and the latter aged 40: interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on the life of the man is, By Table XVI, equal to 12·297, and the value of an annuity on the life of the woman is 14·401; the sum of these is 26·698: from which we must subtract 10·286, the value of an annuity on their joint lives by Table XVIII; and the difference, or 16·412 will be the value of an annuity on the longest of their two lives.

Had the two lives been each of them 10 years older,

or 56 and 50 years of age, then the sum of the values of an annuity on their single lives would, by Table XVI, be equal to 21·766 (that is, equal to 9·717 added to 12·049), and the value of an annuity on their joint lives would, by Table XVIII, be equal to 7·874; consequently 13·892, or the difference between these two values, would be the value of an annuity on the longest of their lives.

Had both the lives been 40 years of age, then 10·964 (or the value of an annuity on their joint lives) subtracted from 28·069 (or the sum of the values of an annuity on their single lives*) would give 17·105 for the answer in this case required.

QUESTION IX.

§ 386. To find the value of an annuity on the *Longest* of *three* lives.

SOLUTION.

From the sum of the values of an annuity on all the single lives, subtract the sum of the values of an annuity on each *pair* of joint lives, and to the difference add the value of an annuity on the three joint lives: this last sum will be the value required. See § 56.

Example. What is the value of an annuity on the longest of three lives aged 20, 30, and 40; in-

* The value of an annuity on the life of the man is 13·668, and the value of an annuity on the life of the woman is 14·401.

terest at 4 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on each single life is, by Table XXVII, equal to 16·033, 14·781, and 13·197 respectively, the sum of which is 44·011; the value of an annuity on each pair of joint lives, (viz 20 and 30, 20 and 40, 30 and 40) is, by Tables XXX and XXXII, equal to 11·873, 10·924, and 10·490 respectively, the sum of which is 33·287; the difference between these two values is 10·724, which being added to 8·986, (or the value of an annuity on the three joint lives, as found by Table XLIV) will give 19·710 for the number of years purchase required.

Had all the three lives been 20 years of age, the value of an annuity on their single lives would have been equal to thrice 16·033, or 48·099; the value of an annuity on each pair of joint lives would have been equal to 37·605, or to thrice 12·535 (that is, equal to thrice the value of an annuity on two joint lives both aged 20, as found by Table XXVIII); and the value of an annuity on the three joint lives would, by Table XLIII, be equal to 10·342: therefore 20·836 would be the number of years purchase in this case required.

QUESTION X.

§ 387. To find the value of an annuity granted upon *three* lives, but to continue only as long as any *two* of them are in being together.

SOLUTION.

From the sum of the values of an annuity on each *pair* of joint lives, subtract twice the value of an annuity on the three joint lives, the difference will be the value required. See § 64.

Example. An annuity is purchased upon three lives aged 20, 30, and 40; on this condition, that as soon as any two of the lives fail, the annuity shall cease: the value of the same is required, reckoning interest at 4 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on each pair of joint lives (viz 20 and 30, 20 and 40, 30 and 40) is, by Table XXX and XXXII, equal to 11·873, 10·924, and 10·490 respectively, the sum of which is 33·287; and the value of an annuity on the three joint lives is 8·986: therefore twice the latter quantity, or 17·972, subtracted from 33·287 will give 15·315 for the number of years purchase required.

Had the ages of all the three lives been 20 years, the value would, in this case, have come out equal to 16·921.

QUESTION XI.

§ 388. To find the value of an annuity, on the *longest* of any number of lives, *Deferred* for any given term.

SOLUTION.

Substitute the values of *deferred* annuities on each single and joint lives, instead of the annuities for the *whole continuance* of those lives, and proceed as in the solutions to the two preceding questions. See § 60.

Example 1. What is the value of an annuity granted on the longest of two lives aged 20 and 40, but which is not to be entered on or enjoyed till after the expiration of 30 years: reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of a deferred annuity for 30 years on a life aged 20 is, by Question VI, equal to 2·272; the value of a similar annuity on a life of 40 is equal to ·784; and the value of a similar annuity on the two joint lives is equal to ·496: therefore if from the sum of the two former, or 3·056, we subtract the latter, the difference, or 2·560, will be the value required.

Example 2. A man and his wife (the former aged 46, and the latter aged 40) purchase on the longest of their two lives the reversion of the lease of an estate, which they are not to enter upon till the end of 10 years: what is the present value of the same, interest being reckoned at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on the life of a male aged 46, deferred 10 years, is, by Question VI, equal to 5·093; the value of a similar annuity on a female

aged 40, is, by the same question, equal to 6·926; and the value of a similar annuity on their joint lives is equal to 3·511. Consequently this latter value deducted from the sum of the two former ones, will leave 8·508 for the answer required.

§ 389. These examples give the present values in a *single* payment; but, if we wish to determine the same value in *annual* payments commencing immediately, we must divide the single payment thus found, by unity added to the value of an annuity on the longest of the given lives for one year less than the given term.

Thus in the second example, the value of the deferred annuity in a *single* payment is 8·508; and, by the rule in the following question,* the value of an annuity on the longest of the two lives for 9 years is equal to 7·251: consequently 8·508 divided by 8·251 will give 1·031 for the value in *annual* payments.

SCHOLIUM.

§ 390. It should here be particularly observed that if the deferred annuity, mentioned in this question, depends upon the *joint* existence of all the lives, to

* A more convenient method however of determining *such* temporary annuities is expressed by the following rule. To the value of the deferred annuity on the longest of the given lives add the expectation that the *longest* of such lives shall receive £1 at the end of the given term; subtract the sum from the value of an annuity on the longest of the given lives: the difference will be the value of the annuity for *one year less than the given term.* See the note in page 350.

the end of the given term, the solution will be materially different; and these two cases must not be confounded. In this latter case the value will be equal to the value of an annuity on the longest of the same number of lives (each older by the given term than the given lives) multiplied by the expectation that the *joint* lives shall receive £1 at the end of that term. See § 61.

Example 1. What is the value of an annuity on the longest of two lives aged 20 and 40, but which is not to be entered upon till the end of 30 years, and then only in case *both* the lives are in existence: interest at $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of an annuity on the longest of two lives aged 50 and 70 is, by the rule in Question VIII, equal to 12·625; and the expectation, that two lives aged 20 and 40 will receive £1 at the end of 30 years, is, by Question II, equal to ·0899: the product of these two quantities will give 1·135 for the answer required.

Example 2. A man (aged 46) and his wife (aged 40) purchase an annuity on the longest of their two lives, which is to commence at the end of 10 years provided they are *both* alive: what is the present value of the same, interest at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of an annuity on the longest of two lives (a man aged 56, and a woman aged 50) is, by the rule in Question VIII, equal to 13·892; and the expectation that two such lives aged 46 and 40 will receive £1 at

the end of 10 years is, by Question II, equal to ·4459: the product of these two quantities will give 6·194 for the value required.

§ 391. The value of these annuities in *annual* payments commencing immediately will be equal to the value in a *single* payment, divided by unity added to the value of an annuity on the *joint* lives for one year less than the given term.

Thus, in the second example, the value of the deferred annuity in a *single* payment is equal to 6·194: and by the rule in the note to page 380, the value of an annuity on the two joint lives deferred for 9 years is 6·329: consequently 6·194 divided by 7·329 will give ·845 for the value in annual payments.

QUESTION XII.

§ 392. To find the value of a *Temporary* annuity on the *longest* of any number of lives.

SOLUTION.

From the absolute value of an annuity on the longest of the given lives, subtract the value of the same annuity *deferred* during the given term: the difference will be the value required.* See § 62.

* Or (which is the same thing) substitute the values of *temporary* annuities on each single and joint lives, instead of the values of annuities for the *whole continuance* of those lives; and proceed as in the solutions to Questions VIII and IX.

Example 1. What is the value of a temporary annuity for 30 years on the longest of two lives aged 20 and 40; reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of life as given by *M. De Parcieux?*

The value of an annuity on the longest of those lives is, by Question VIII, equal to 18·333; and the value of an annuity on the longest of those lives *deferred* for 30 years is, by Question XI, equal to 2·560; consequently the difference between these two values, or 15·773, will be the answer required.

Example 2. A man aged 46 purchases an annuity for 10 years, terminable, however, at any time prior thereto, on the extinction of his own life and the life of his wife aged 40; what is the value of the same, interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on the longest of their lives is, by Question VIII, equal to 16·412; and the value of an annuity on the longest of their lives *deferred* for 10 years is, by Question XI, equal to 8·508; the difference therefore between these two values, or 7·904, is the answer required.

QUESTION XIII.

§ 393. To find the value of the *Reversion* of an annuity on a *single* life after any other *single* life.*

* This Question, and also Question XVIII, are of considerable importance in enabling us to determine the best means of providing Annuities for the *benefit of Widows;* as will be more fully explained in the following Chapter.

SOLUTION.

From the value of an annuity on the life in reversion, subtract the value of an annuity on the two joint lives; the difference will be the value required. See § 76.

Example 1. A person aged 20 wishes to purchase an annuity for what may happen to remain of his life beyond another life aged 40: what ought he to give for the same, allowing interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on the life in reversion (that is, on the life aged 20) is, by Table V, equal to 16·624; and the value of an annuity on the two joint lives is, by Table IX, equal to 12·545; therefore, the difference of these two values, or 4·079, is the number of years purchase required.

Had the life in reversion been 40, and the life in possession 20 years of age, the value would have come out equal to 1·709.

Or, had both the lives been 20 years, the value would have come out equal to 2·620: and had they both been 40 years of age, the value would have come out equal to 2·544.

Example 2. What is the value of an annuity to be enjoyed by a woman aged 40, during her life, after the decease of her husband aged 46; interest at 4 per cent, and the probabilities of living as amongst males and females respectively in *Sweden?*

The value of an annuity on the life of a woman

aged 40 is, by Table XVI, equal to 14·401, and the value of an annuity on their joint lives is, by Table XVIII, equal to 10·286; therefore 4·115 is the number of years purchase required.

Had their lives been both 40 years of age, the value would have come out equal to 3·437.*

SCHOLIUM.

§ 394. If, instead of a *single* payment, we wish to determine the value of these reversionary annuities in *annual* payments to be made during the existence of the two joint lives; we must divide the value, found in either case, by unity added to the value of an annuity on the joint lives: and the quotient will give the annual payments required.

Thus, in the first example, 4·079 being divided by 13·545 will give ·301 for the *annual* payments which ought to be made during the joint lives, as an equivalent for the sum in a single payment. In like manner, in the second example, 4·115 being divided by 11·286 will give ·365 for the *annual* payments which ought to be made by a man aged 46 during the joint lives of himself and his wife, aged 40, in order to secure

* It is worthy of remark that the value of a reversionary annuity on one life after another is, when the difference of age is not very considerable, *less* in the younger ages and *greatest* in the middle ages of life: a circumstance which may be attributed to the higher chances of living in the younger ages, whereby the probability of *survivorship* is deferred so long as to affect in a material degree the value of the *reversionary* annuity.

to his widow, on his death, an annuity of £1 per annum during her life.*

QUESTION XIV.

§ 395. To find the value of the *Reversion* of an annuity on a *single* life A, after the *longest* of two other lives, P and Q.

SOLUTION.

From the sum of the values of an annuity on the single life A in reversion, and on the three joint lives, subtract the sum of the values of an annuity on the two joint lives AP and AQ: the difference will be the value required. See § 76.

Example. What is the value of an annuity on the life of a person aged 20 to be enjoyed by him after the decease of both his brother and sister, aged 30 and 40 respectively: interest at 4 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on the single life in reversion is, by Table XXVII, equal to 16·033, and the

* Dr. Price has given a table of the value of reversionary annuities for the life of a wife after the death of her husband; both in *single* and *annual* payments during their joint lives: deduced from the *Sweden* observations and at 4 per cent interest, according to the several ages therein mentioned. See his *Obs. on Rev. Pay.* vol. ii. page 431. The utility and convenience of the present rule, in enabling us to determine the propriety and efficacy of those schemes which are instituted for the *benefit of Widows*, will be shown in the following Chapter.

value of an annuity on the three joint lives is, by Table XLIV, equal to 8·986, the sum of which is 25·019; the value of an annuity on the two joint lives 20 and 30 is, by Table XXX, equal to 11·873, and the value of an annuity on the two joint lives 20 and 40 is, by Table XXXII, equal to 10·924, the sum of which is 22·797; therefore 22·797 subtracted from 25·019 will leave 2·222 for the value required.

Had the two lives in possession been both 40, then 16·033 added to 8·601 (or the value of an annuity on three joint lives aged 20, 40 and 40, as found by Question V) will make 24·634; from which we must subtract twice 10·924: the difference, or 2·786, will be the value in this case required.

QUESTION XV.

§ 396. To find the value of the *Reversion* of an annuity on the *longest* of two lives A and B, after any *single* life P.

SOLUTION.

From the sum of the values of an annuity on each single life A and B in reversion and on the three joint lives, subtract the sum of the values of an annuity on each pair of joint lives AB, AP, BP: the difference will be the value required. See § 76.

Example. What is the value of an annuity on the longest of two lives aged 20 and 30, to be enjoyed after the extinction of a single life aged 40; interest at 4 per cent, and the probabilities of living as at *Northampton?*

By proceeding as in the last question it will be found that the sum of the values of an annuity on the two single lives 20 and 30 is 30·814, that the value of an annuity on the three joint lives is 8·986, and that the sum of the values of an annuity on each pair of joint lives is 33·287; consequently, 6·513 is the value required.

Had the two lives in reversion been both 20 years of age, then the sum of the values of an annuity on their single lives would be 32·066; the value of an annuity on the three joint lives would, by Question V, be 9·406; and the sum of the values of an annuity on each pair of joint lives would be 34·383: consequently 7·089 would be the value required.

On the Renewal of Leases for Lives.

§ 397. The three preceding questions will be found of great practical use in the *Renewal of Leases** held on two or three lives; as they serve to show the value of the *Fine* that ought to be paid for *putting in* a new life in lieu of one that has *dropt*, or become extinct. For, the value† of such fine is equal to the

* See more on the subject of the Renewal of Leases for lives and *afterwards for a term certain*, in the observations at the end of Question XXIV.

† I call the *value* of a Fine, the *Number of years purchase* that it is worth: agreeably to the principles laid down in deducing the value of annuities: see the remark in the note in page 362. This value, being multiplied by the net improved rent of the estate, will show the total sum of money that ought to be given for the renewal.

difference between the value of an annuity on the longest of all the lives (including the life or lives to be added) and the value of an annuity on the longest of the lives in possession: which rule will be found to agree with the solutions above given, according to the several cases there mentioned.*

Example 1. The value of the fine which ought to be given for *putting in* a new life, aged 20, to a lease held by Two lives, after One has dropped, is (supposing the existing life to be aged 40) equal to 4·079, or rather more than 4 years purchase of the net improved rent of the estate;† as already found by Question XIII.

Consequently, if the net improved rent of the estate had been £100 per annum, we should have £407 : 18 for the gross *sum* that ought to be paid down for the renewal required.

Example 2. The value of the fine which ought to be given for *putting in* a new life, aged 20, to a lease held by Three lives, after One has dropped, is (supposing the existing lives to be aged 30 and 40) equal to 2·222, or nearly $2\frac{1}{4}$ years purchase of the net improved rent of the estate; as already found by Question XIV.

Example 3. The value of the fine which ought to be given for *putting in* Two new lives, both aged 20,

* This subject is more fully discussed in my *Tables for the Purchasing and Renewing of Leases*, 2d edit. 1807.

† That is, the net *surplus* rent, after deducting the reserved rent (if any) and all taxes and other annual charges.

to a lease held by Three lives, after Two have dropped, is (suppoing the existing life to be aged 40) equal to 7·089, or rather more than 7 years purchase of the net improved rent of the estate; as already found by Question XV.

The same principles will also lead us to the true values that ought to be given for *Exchanging* any one or more lives (in a lease) for a life or lives of any other age. For, such value will in all cases be equal to the present value of the tenant's interest in the lease *before* the exchange, subtracted from his interest in the lease *after* the new lives are added.

QUESTION XVI.

§ 398. To find the value of the *Reversion* of an annuity on a *single* life, after the extinction of two *joint* lives.

SOLUTION.

From the value of an annuity on the single life in reversion, subtract the value of an annuity on the three joint lives: the difference will be the value required. See § 76.

Example. What is the value of an annuity on the life of a person aged 20 to be enjoyed by him after the decease of either of his brothers, one aged 30 and the other 40; interest at 4 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on the life of a person

Example 1. What is the value of a temporary annuity for 30 years on the longest of two lives aged 20 and 40; reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of life as given by *M. De Parcieux?*

The value of an annuity on the longest of those lives is, by Question VIII, equal to 18·333; and the value of an annuity on the longest of those lives *deferred* for 30 years is, by Question XI, equal to 2·560; consequently the difference between these two values, or 15·773, will be the answer required.

Example 2. A man aged 46 purchases an annuity for 10 years, terminable, however, at any time prior thereto, on the extinction of his own life and the life of his wife aged 40; what is the value of the same, interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on the longest of their lives is, by Question VIII, equal to 16·412; and the value of an annuity on the longest of their lives *deferred* for 10 years is, by Question XI, equal to 8·508; the difference therefore between these two values, or 7·904, is the answer required.

QUESTION XIII.

§ 393. To find the value of the *Reversion* of an annuity on a *single* life after any other *single* life.*

* This Question, and also Question XVIII, are of considerable importance in enabling us to determine the best means of providing Annuities for the *benefit of Widows;* as will be more fully explained in the following Chapter.

SOLUTION.

From the value of an annuity on the life in reversion, subtract the value of an annuity on the two joint lives; the difference will be the value required. See § 76.

Example 1. A person aged 20 wishes to purchase an annuity for what may happen to remain of his life beyond another life aged 40: what ought he to give for the same, allowing interest at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on the life in reversion (that is, on the life aged 20) is, by Table V, equal to 16·624; and the value of an annuity on the two joint lives is, by Table IX, equal to 12·545; therefore, the difference of these two values, or 4·079, is the number of years purchase required.

Had the life in reversion been 40, and the life in possession 20 years of age, the value would have come out equal to 1·709.

Or, had both the lives been 20 years, the value would have come out equal to 2·620: and had they both been 40 years of age, the value would have come out equal to 2·544.

Example 2. What is the value of an annuity to be enjoyed by a woman aged 40, during her life, after the decease of her husband aged 46; interest at 4 per cent, and the probabilities of living as amongst males and females respectively in *Sweden?*

The value of an annuity on the life of a woman

equal to ·496. Therefore the difference between these two values, or 1·776, will, by Question XIII, be the answer required.

Example 2. A woman aged 40 will at the end of 10 years enter upon an annuity for her life, provided her husband, now aged 46, be then dead; or if this should not be the case, then at the end of any year in which he may die: what is the present value of the reversion, interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity on the life of a female aged 40, deferred for 10 years, is, by Question VI, equal to 6·926; and the value of an annuity on their joint lives is, by the same Question, equal to 3·511. Therefore the difference between these two values, or 3·415, will, by Question XIII, be the answer required.

If we wish to determine the value of these deferred reversionary annuities in *annual* payments during the continuance of their joint lives, we have only to divide the *single* payment, above found, by unity added to the value of an annuity on the joint lives; as already explained in the Scholium to Question XIII.

SCHOLIUM.

§ 401. If the deferred annuity mentioned in this question depends on the *joint* continuance of all the lives to the end of the given term, the solution will be materially different, (as I have already observed respecting deferred annuities depending on the longest of any lives, in the Scholium in page 389): and care

must be taken not to confound the two cases together. When the reversion depends on the *joint* continuance of all the lives to the end of the given term, its value will be equal to the value of the reversion on the same number of lives, each older by the given term than the given lives, multiplied by the expectation that the *joint* lives shall receive £1 at the end of that term. See § 78.

Example 1. What is the present value of a reversionary annuity on a life aged 20 for what may happen to remain of it beyond another life aged 40 after 30 years, provided *both* lives continue from the present time to the end of the term: interest at $4\frac{1}{2}$ per cent, and the probabilities of living as given by *M. De Parcieux?*

The value of an annuity on a life aged 50 after another life aged 70 is, by the rule in Question XIII, equal to 6·404 (or the difference between 11·921 and 5·517); and the expectation that the joint lives, 20 and 40, shall receive £1 at the end of 30 years is, by Question II, equal to ·0899; the product of these two quantities therefore, or ·576, will be the value required.

Example 2. What is the present value of an annuity on the life of a woman aged 40, for what may happen to remain of it beyond the life of her husband, now aged 46, after 10 years, provided they *both* continue in being so long: interest at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of an annuity on the life of a woman

aged 50 after the decease of her husband aged 56, is, by the rule in Question XIII, equal to 4·175; and the expectation that the joint lives 40 and 46 shall receive £1 at the end of 10 years is, by Question II, equal to ·4459: consequently, these two quantities multiplied together will produce 1·862 for the value in this case required.

If we wish to determine the value of these deferred reversionary annuities in *annual* payments during the continuance of their joint lives, we have only to divide the *single* payment, above found, by unity added to the value of an annuity on the joint lives; as already explained in the Scholium to Question XIII.

QUESTION XIX.

§ 402. To find the value of any *Temporary reversionary* life annuity.

SOLUTION.

Substitute the values of *temporary* annuities on each single and joint lives, instead of the annuities for the *whole continuance* of those lives; and proceed as in the last question. See § 77.

Example 1. A lease of an estate is held for 30 years, to the rent of which a person now aged 20 will be entitled on the decease of his brother aged 40: what is the value of his interest therein, taking the

probabilities of life as observed by *M. De Parcieux*, and interest at $4\frac{1}{2}$ per cent?

The value of an annuity for 30 years on a life aged 20 is, by Question VII, equal to 14·352, and the value of a similar annuity on the two joint lives is, by the same Question, equal to 12·049; consequently, the difference of these two values, or 2·303, will, by Question XIII, be the answer required.

Example 2. In a lease of an estate (originally granted for twenty-one years) 10 years are unexpired; to the rent of which, a woman aged 40 will, on the decease of her husband aged 46, become entitled: what is the value of her interest in the same, taking the probabilities of life as observed in *Sweden*, and the rate of interest at 4 per cent?

The value of an annuity on the life of a woman aged 40 for 10 years is, by Question VII, equal to 7·475, and the value of a similar annuity on the two joint lives is, by the same Question, equal to 6·775: consequently ·700 is the value required.

If we wish to determine the value of these temporary reversionary annuities in *annual* payments, we have only to divide the single payments, above found, by unity added to the value of a temporary annuity on the joint lives *for one year less than the given term*: and the quotient will be the annual payment required. See § 368.

QUESTION XX.

§ 403. Two persons A and B purchase an an-

nuity on the longest of their lives, which is to be equally divided between them whilst they are both living, but on the decease of either of them it is to belong wholly to the *Survivor:* to find their respective shares, or the proportion which each person ought to contribute towards the purchase.

SOLUTION.

From the value of an annuity on the life A or B subtract half the value of an annuity on the two joint lives: the remainder will be the share of A or B required. See § 85.

Example 1. Suppose the age of A to be 20, and that of B 40; the rate of interest $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of an annuity on a life aged 20 is, by Table V, equal to 16·624; and the value of an annuity on a life aged 40 is equal to 14·254: also the value of an annuity on two joint lives aged 20 and 40 is, by Table IX, equal to 12·545, the half of which is 6·272. Consequently this latter value subtracted from 16·624 will give 10·352 for the share which A ought to contribute; and subtracted from 14·254 it will give 7·982 for the share which B ought to contribute.

Example 2. Suppose two persons, a man aged 46 and a woman aged 40, to hold the lease of an estate on the longest of their lives, the rent of which is di-

vided in the manner above stated; what sum ought to be given to each of them for surrendering their right in the same: interest being reckoned at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of the man's interest is equal to 7·154 (or to the difference between 12·297 and 5·143); and the value of the woman's interest is equal to 9·258 (or to the difference between 14·401 and 5·143). Therefore if the net rent of the estate were £50 per annum, the sum which ought to be given to the man will be 357·700, or £357 : 14 : and the sum which ought to be given to the woman will be 462·900, or £462 : 18.

SCHOLIUM.

§ 404. If the annuity is for a term of years *less* than that to which it is probable the given lives may extend, we must substitute the values of annuities for the given *term* instead of the values of annuities for the *whole continuance* of the lives: and proceed with these substituted values according to the directions given in the rule.

Thus, if in the first example, the annuity had been for 30 years only, we must find the value of a *temporary* annuity for 30 years on a single life aged 20, a single life aged 40, and two joint lives aged 20 and 40: which values are, by Question VII, equal to 14·352, 13·470, and 12·049 respectively. Consequently, the half of the latter subtracted from 14·352

will leave 8·328 for the share of A; and subtracted from 13·470, it will leave 7·446 for the share of B.

QUESTION XXI.

§ 405. Two persons are in possession of an annuity on the longest of their lives; which, on the decease of either of them, will belong to D and his heirs during the life of the *Survivor:* to find the value of his interest therein.

SOLUTION.

From the sum of the values of an annuity on each single life in possession, subtract twice the value of an annuity on their joint lives: the difference will be the value required. See § 137.

Example. Suppose the ages of the two lives in possession to be 20 and 40; interest 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on each single life is, by Table V, equal to 16·624 and 14·254; and the value of an annuity on the two joint lives is, by Table IX, equal to 12·545. Consequently, 25·090 subtracted from 30·878 will leave 5·788 for the interest of D and his heirs in this annuity.

Had the two lives in possession been a man aged 46 and a woman aged 40, the value of the interest of

D and his heirs in the annuity would (on the supposition that interest was at 4 per cent, and the probabilities of living as observed in *Sweden*) be equal to 6·126.

SCHOLIUM.

§ 406. If the annuity is for a term of years *less* than that to which it is possible that either of the given lives may extend, we must substitute the values of annuities for the given *term* instead of the values of annuities for the *whole continuance* of the lives: and proceed with these substituted values according to the directions given in the rule.

Thus, if in the example just given, the annuity on the two lives aged 20 and 40 had been for 30 years only, then 24·098 (or twice the value of a temporary annuity for 30 years on the two joint lives, as found by Question VII) subtracted from 27·822 (or the sum of the values of a temporary annuity for 30 years on each of the single lives, as found by the same Question) will leave 3·724 for the value in this case required.

QUESTION XXII.

§ 407. To find the value of an annuity *certain* for a given term; and afterwards, for the *remainder* of any given life or lives.

SOLUTION.

To the present value of an annuity certain for the

given term, add the value of an annuity on the given life, or lives, *deferred* for that term: the sum of these two will be the value required. See § 51.

Example 1. What is the value of an annuity *certain* for 30 years, and then to continue during the life of a person now aged 20: reckoning interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity certain for 30 years is, by Table LIX, equal to 16·289, and the value of an annuity, on a life aged 20, deferred for 30 years is, by Question VI, equal to 2·272: consequently 18·561 will be the value required.

Example 2. What is the value of an annuity *certain* for 10 years, and then to continue during the joint lives of a man aged 46 and his wife aged 40: reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden?*

The value of an annuity certain for 10 years is, by Table LIX, equal to 8·111, and the value of an annuity on the two joint lives, deferred for 10 years, is, by Question VI, equal to 3·511: consequently 11·622 will be the value required.

QUESTION XXIII.

§ 408. Supposing a person to enjoy an annuity for his life; and, at his decease, to have the nomination of a successor: to find the present value of the annuity on the *Succeeding life.*

SOLUTION.

Multiply the value of an annuity on the life in possession by the rate of interest, and subtract the product from unity; multiply the remainder by the assumed* value of an annuity on the succeeding life: the product will be the present value required. See § 205.

Example. Suppose the age of the life in possession to be 65, and that at his decease he has the liberty of nominating another life to succeed him; which life we will suppose to be one of the best that can *then* be found, or one which may *then* be about 10 years old: what is the *present* value of such succeeding life, interest at 5 per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on a life aged 65 is, by Table V, equal to 7·535, which being multiplied by ·05, and subtracted from unity, will leave ·62325; and this quantity, multiplied by 16·213 (or the value of an annuity on the life to be hereafter nominated) will produce 10·105 for the *present* value of the same, as was required.

Had it been required to calculate the value of the

* The life or lives nominated to succeed to the annuity, after the extinction of the life or lives in possession, are such as are *then* to be fixed on, at pleasure: therefore the *present* value of an annuity on those lives will vary according to the ages at which they are supposed to be put in. See page 163.

succeeding life according to the *Northampton* tables of observations, at the same rate of interest; we ought to multiply 7·276 by ·05; which, subtracted from unity, would leave ·63620; and this latter quantity multiplied by 15·139* will give 9·631 for the value in this case required.

SCHOLIUM.

§ 409. The solution given to the present question will apply equally to the case of annuities on *joint* lives, or on the *longest* of any lives; with power to nominate, at the extinction of such lives, an equal number of similar lives to succeed thereto.

Example. Suppose an annuity is held on two *joint* lives aged 50 and 60; and that, on the extinction of either of them, two other joint lives (the best that can then be found, and which we will suppose to be 10 years old) are nominated to succeed them: what is the present value of the annuity on the succeeding joint lives, interest at 5 per cent, and the probabilities of living as observed at *Northampton?*

* It appears that, by the *Northampton* tables, a life of the age of 8 years is one of the best that can be put in; since the value of an annuity on such life is equal to 15·226: but in questions of this kind we may safely omit the *decimal,* and assume the life to be such that an annuity on it at the time of nomination would be worth 15 years purchase. For, it seldom happens that the life, which we should *choose* to nominate, is exactly of the age which is assumed.

The value of an annuity on the two *joint* lives is, by Table XXX, equal to 6·568; which, being multiplied by ·05 and subtracted from unity, will leave ·67160; and this multiplied by 12·665 (or the value of an annuity on two joint lives both 10 years old) will produce 8·506 for the value required.

Had the annuity been held on the *longest* of the two lives, aged 50 and 60; with power, on the extinction of *both* those lives, to nominate two other lives (whose ages we will suppose to be each 10 years) who are to enjoy the annuity as long as either of them is in existence; the present value of the annuity on those succeeding lives might be calculated in a similar manner. For, the value of an annuity on the longest of two lives aged 50 and 60 is, by the solution in Question VIII, equal to 12·093; which being multiplied by ·05 and subtracted from unity, leaves ·39535; and this quantity multiplied by 17·613 (or the value of an annuity on the longest of two lives aged 10 years) will give 6·963 for the value required.

QUESTION XXIV.

§ 410. To find the present value of an annuity *certain* for a given term *after* the extinction of any given life or lives.

SOLUTION.

Multiply the value of an annuity on the given life or lives by the rate of interest, and subtract the pro-

duct from unity; multiply the remainder by the present value of an annuity certain for the given term: the product will be the value required. See § 208.

Example 1. Suppose D and his heirs to be entitled to an annuity certain for 21 years, to commence at the death of a person aged 70; what is the present value of D's interest in that annuity, taking the probabilities of living as observed at *Northampton*, and the rate of interest at 5 per cent?

The value of an annuity on the life of a person aged 70 is, by Table XXVII, equal to 6·023, which being multiplied by ·05 and subtracted from unity, will leave ·69885; and this multiplied by 12·821 (or the value of an annuity certain for 21 years) will produce 8·960 for the present value of the same annuity to be entered on* at the extinction of the given life.

If this value be added to 6·023 (or the value of the annuity on the life in possession) the sum of them, or 14·983, will be equal to the value of an annuity on the given life, commencing immediately, and to continue, after the extinction of such life, for the term of 21 years longer.

Example 2. A lease of an estate is held upon two lives aged 60 and 70; and, after the decease of both of them, then for 21 years longer: what is the value

* This solution supposes that the first payment of the annuity is made at the end of the year in which the given life becomes extinct.

of such lease, reckoning interest at 5 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on the longest of two lives aged 60 and 70 is, by the rule in Question VIII, equal to 9·735; which, being multiplied by ·05 and subtracted from unity, will leave ·51325; this being multiplied by 12·821 (or the value of an annuity certain for 21 years) will give 6·580 for the present value of the same annuity to be enjoyed 21 years after the extinction of the longest of the two lives. And this value being added to 9·735 (or the value of an annuity on the longest of the two lives) will give 16·315 for the value of the lease required.

Example 3. A lease of an estate is held upon three lives aged 50, 60, and 70; and, after their decease, then for 21 years longer: what is the value of the same, reckoning interest at 5 per cent, and the probabilities of living as at *Northampton?*

The value of an annuity on the longest of three lives aged 50, 60, and 70 is, by the rule in Question IX, equal to 12·443;* which, being multiplied by ·05 and then subtracted from unity, will leave ·37785; this being multiplied by 12·821 will give 4·844 for the present value of the annuity for 21 years after the longest of the three lives. And this value, being added to 12·443, will give 17·287 for the value of the lease required.

* The value of an annuity on the three *joint* lives is, by the rule in Question V (and the correction alluded to in page 376) equal to 4·061.

Had the three lives been 10, 60, and 70 years of age; then the value of an annuity on the longest of their lives would be equal to 15·878;* and the value of the annuity for 21 years after those lives would be equal to 2·642: consequently the value of the lease would in this case be equal to 18·520.

Or, had the three lives been 10, 10 and 70 years of age; then the value of an annuity on the longest of their lives would be equal to 17·610;† and the value of the annuity for 21 years after those lives would be equal to 1·532: consequently the value of the lease would in this case be equal to 19·142.

On the Renewal of Leases for Lives and afterwards for a Term certain.

§ 411. The three examples given in the preceding question will serve to show the method of determining the value of the *Fine* which ought to be given for *Renewing* any lives dropt in a lease originally held on three lives *and for a term certain after the extinction of those lives.* For, the value of such fine will in all cases be equal to the present value of the tenant's interest in the lease *before* the renewal, subtracted

* The value of an annuity on these three *joint* lives is, by the rule in Question V (and the correction alluded to in page 376) equal to 4·454.

† The value of an annuity on these three *joint* lives is, by the rule in Question V (and the correction alluded to in page 376) equal to 5·374.

from his interest in the lease *after* the new lives are added.*

Thus, suppose that, in a lease originally held on three lives and 21 years, *one* of the lives has dropt; and that the ages of the two remaining lives are 60 and 70: the value of the fine which ought to be paid for putting in another life aged 10 years is equal to the difference between 16·315 (or the value found by the second example) and 18·520 (or the value found by the second case in the third example). That is, the value of the fine will be equal to 2·205, or about $2\frac{1}{5}$ years purchase of the net improved rent of the estate.

Again, let us suppose that *two* of the lives have dropt; and that the age of the remaining life is 70: the value of the fine which ought to be paid for putting in two other lives both 10 years of age, is equal to the difference between 14·983 (or the value found by the first example) and 19·142 (or the value found by the third case in the third example): consequently the value of the fine will be equal to 4·159, or about $4\frac{1}{6}$ years purchase of the net improved rent of the estate.

These examples will also serve to show the sum that ought to be given for *Exchanging* any of the lives on which the lease may happen to be held: for, the same method of solution will apply to such cases.

* See what has been already said on the subject of the Renewal of Leases for Lives in general, in page 397.

Thus, suppose that, in a lease held on three lives and 21 years, the ages of the lives at present in the lease were 50, 60, and 70; and that the tenant is desirous of exchanging the life of 50 for another life aged 10 years old: the value of the fine which ought in such case to be paid will be equal to the difference between 17·287 (or the value of his *present* interest, as found by the first case in the third example) and 18·520 (or the value of his interest *after* the exchange, as found by the second case in the same example). That is, the value of the fine will be 1·233, or near $1\frac{1}{4}$ year's purchase of the net improved rent of the estate.

§ 412. Many of the estates belonging to the Corporation of *Liverpool* are held on the tenure alluded to in these examples; and till lately they were in the constant habit of renewing their leases on the following terms: viz. One year's purchase for adding *one* life dropt, Three years purchase for adding *two* lives dropt, and Seven years purchase for adding *three* lives dropt when the 21 years remain unexpired. In all these cases no regard was paid to the *age* or state of health of the existing lives in the lease. This practice of demanding an uniform fine for renewing with *any* life, and without regard to the age or state of health of the lives remaining in the lease, betrayed a total want of knowledge on the subject; and was in most cases injurious to the interests of the Corporation.

But, the most singular circumstance attending this subject was their custom of *exchanging*, for the sum

of only *one guinea* each, lives, not exceeding 50 years of age and in good health, for lives of *any other age*, and in estates of *any* yearly value!!! A practice which could hardly be supposed ever to have existed in so enlightened a place as *Liverpool*. The Corporation at length, suspecting that their mode of proceeding was incorrect in principle, referred the matter to a Committee, who directed it to be laid before me for my opinion: and agreeably to their request I calculated a set of Tables for their use, founded on the principles detailed in the preceding examples.

As it is probable that many other Corporate bodies are still pursuing the same incorrect and absurd practice of leasing their estates, I have been more particular in these examples, in order that they may the more readily determine the values that *ought* to be given in such cases.

QUESTION XXV.

§ 413. To find the present value of what may happen to *remain* (after any given life, or lives) of an annuity *certain* for a given term: provided such term be *less* than that to which it is possible the given lives may extend.*

* The term to which it is possible that any given life or lives may extend is—for a *single* life, equal to the difference between the age of such life and the age of the oldest life in the table of observations:—for *joint* lives, equal to the difference between the *oldest* of such lives and the age of the oldest life in the table of

SOLUTION.

From the value of an annuity certain for the given term, subtract the value of an annuity on the given life, or lives, *for the given term:* the difference will be the value required. See § 192.

Example. A lease of an estate is held for 30 years, to the rent of which a person aged 20 is entitled provided he lives so long; but if not, then the remainder of the lease will descend to his heirs: what is the value of their interest in the same, taking the probabilities of living as observed by *M. De Parcieux*, and reckoning interest at the rate of $4\frac{1}{2}$ per cent?

The value of an annuity certain for 30 years is, by Table LIX, equal to 16·289; and the value of a temporary annuity for 30 years on a life aged 20 is, by Question VII, equal to 14·352: therefore this latter quantity subtracted from the former will leave 1·937 for the value of the reversion required.

Had the life been 40 years of age, the value would have been equal to the difference between 16·289 and 13·470: that is, 2·819 would be the value of the reversion in this case required.

Had these two *joint* lives (aged 20 and 40) been entitled to the rent of the estate provided they lived so

observations:—and for the *longest* of any number of lives, equal to the difference between the *youngest* of such lives and the age of the oldest life in the table of observations.

long, then 12·049 (or the value of a temporary annuity for 30 years on those two *joint* lives, as found by Question VII) subtracted from 16·289, would give 4·240 for the value of the reversion in this case required.

Or, had the *longest* of these two lives been entitled to the rent of the estate, then 15·773 (or the value of a temporary annuity for 30 years on the *longest* of those lives, as found by Question XII) subtracted from 16·289, would give ·516 for the value of the reversion in this case required.

QUESTION XXVI.

§ 414. To find the value of the *Assurance* of an *estate* (or annuity certain for any given term*) to be entered upon at the extinction† of any given lives.

SOLUTION.

Subtract the value of an annuity on the given lives‡ from the value of the perpetuity, or the terminable

* Provided such term be *not less* than that to which it is probable the given lives may extend. For, in *such* case, the solution is obtained by the preceding Question.

† That is, the first payment of the annuity is to be made at the end of the year in which such lives become extinct: and this is always understood in questions of this kind.

‡ Whether a *single* life, or *joint* lives, or the *longest* of any number of lives: for the solution will apply to each case.

annuity; and the difference will be the value required. See § 189.

Example 1. What is the value of the reversion of an estate in fee after the death of a person now aged 20; interest being reckoned at $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of the perpetuity is, by Table LIX, equal to 22·222, and the value of an annuity on the life of a person aged 20 is, by Table V, equal to 16·624; consequently the difference between these two values, or 5·598, will be the answer required. Therefore if the estate produced a rent of £4 : 10 per annum, its present value in a *single* payment would be 25·191, or £25 : 3 : 10.

This is the true present value of the assurance in a *single* payment: but, in order to obtain the value of the same in *annual* payments, commencing immediately, we must divide the sum, thus found, by unity added to the value of an annuity on the given life (agreeably to the principles laid down in § 369): and the quotient will be the answer required.

Thus, in the present case, if we divide 25·191 by 17·624, the quotient will be 1·429, or £1 : 8 : 7; and this is the sum that ought to be paid annually during the life of the person assured, in order to secure the perpetuity of £4 : 10 per annum on his death: the first of those annual payments being made *immediately*, and the remaining ones at the *beginning* of every subsequent year.

Had the question referred to an *annuity* for 80 years,* instead of a perpetuity, then 16·624 subtracted from 21·565 (or the present value of an annuity certain for that term by Table LIX) would leave 4·941 for the answer required. Therefore, if the annuity, as in the preceding case, were £4 : 10 per annum, its present value in a *single* payment would be 22·234 : and this sum, divided by 17·624, would give 1·262 for the value of the same in *annual* payments.

Example 2. What is the value of a freehold estate to be entered upon at the death of *either* of two lives, a man aged 46 and a woman aged 40; reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of the perpetuity is, by Table LIX, equal to 25, and the value of an annuity on the joint lives of these two persons is, by Table XVI, equal to 10·286: this latter value subtracted from the former will leave 14·714 for the answer required. Therefore if the estate produced £4 per annum, its present value would be 58·856, or £58 : 17 : 1, in a *single* payment: and this sum, divided by 11·286, would give 5·215, or £5 : 4 : 4, for the value of the same in *annual* payments.

If the estate were not to be entered upon till the

* It must be particularly observed that, when we have to determine by this rule the value of the reversion of any terminable annuity after any given lives, the number of years during which such annuity is to continue must not be *less* than that to which it is probable the given lives may extend: see the note in page 419.

extinction of *both* the lives, then 16·412 (or the value of an annuity on the *longest* of the two lives, as found by Question VIII) subtracted from 25, will leave 8·588 for the number of years purchase required; and which being multiplied by 4, as in the preceding case, will give 34·352 for the value of the same estate in a *single* payment: and this sum divided by 17·412 will give 1·973 for the value of the same in *annual* payments.

Had it been a leasehold estate of £4 per annum for 60 years, instead of a freehold, the value would in the former case have come out equal to 49·348 in a *single* payment; or 4·373 in *annual* payments. And in the latter case, to 24·844 in a *single* payment; or 1·427 in *annual* payments.

QUESTION XXVII.

§ 415. To find the value of an *Assurance* of a given *sum*, which is to be received on the extinction* of any given lives.

SOLUTION.

Multiply the value of an annuity on the given lives†

* That is, at the end of the year in which such lives become extinct: and this is always understood in questions of this kind. The usual practice of the Assurance Offices, however, is to pay the sum at the end of six months from the time of the decease.

† Whether a *single* life, or *joint* lives, or the *longest* of any number of lives: for, the solution (as in the preceding question) will apply to each case.

by the rate of interest, and subtract the product from unity; divide the remainder by the amount of £1 in one year: and the quotient, multiplied by the given sum, will be the value required. See § 180.

Example 1. What is the present value of an assurance of £100 on the life of a person aged 20: interest at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on such life is, by Table V, equal to 16·624, which being multiplied by ·045 (or the rate of interest) will produce ·74808; the difference between this value and unity is ·25192, which being divided by 1·045 (or the amount of £1 in one year) will give ·24107 for the present value of £1 to be received on the extinction of the given life:* and this value, being multiplied by 100, will give 24·107, or £24 : 2 : 2, for the answer required, in a *single* payment.

But, in order to obtain the value of the same in

* From the present value of *one pound* to be received on the extinction of any given life or lives, we may readily determine the sum, which ought to be paid, on the extinction of such lives, for *any given sum now advanced:* viz. by dividing this latter sum by the present value of £1 as above found. Thus, if a person, aged 20, borrows £4000, and gives security to pay the value of the same at his death, the sum which ought *then* to be paid (supposing the interest &c the same as mentioned in the text) is found by dividing 4000 by ·24107; which gives 16592·691, or £16592 : 13 : 10, for the answer required. This method is universal; and will apply to all the subsequent questions in the present Chapter.

annual payments, commencing immediately, we must divide the sum, thus found, by unity added to the value of an annuity on the given life; agreeably to the principles laid down in § 369: whence 24·107 divided by 17·624 will give 1·368, or £1 : 7 : 4, for the sum which ought to be paid annually during the life of the person assured, in order to secure the sum of £100 on his death. The first of those annual payments being made *immediately*, and the remaining ones at the *beginning* of every subsequent year.*

Had the life been 40 years of age, its value in a *single* payment would have been equal to 34·318: which being divided by 15·254 (or unity added to the value of an annuity on the given life) will give 2·249, or £2 : 5 : 0, for the value of the same in *annual* payments.

Example 2. What is the present value of £100 to be received on the death of a man aged 46: interest being reckoned at 4 per cent, and the probabilities of life as observed in *Sweden?*

* The rates of Assurances for Lives, at *all* the different Offices established in London, are calculated from the *Northampton* Table of Observations, and at 3 per cent interest. These rates, both for *Single* and *Joint* lives, are given in Tables LI and LII at the end of the present work: and by comparing them with the true and proper values, as given in Tables LIV to LVII, the public may form a tolerably accurate idea of the immense profit which is made by the several Assurance Companies above alluded to. I shall however enlarge more upon this subject in Chapter XIV. See also the Scholium to Question XXIX, and the Scholium to Question XXX.

The value of an annuity on the life of a man aged 46 is, by Table XVI, equal to 12·297; which, being multiplied by ·04, and the product subtracted from unity, leaves ·50812: this quantity, divided by 1·04, gives ·48858; which, being multiplied by 100, produces 48·858, or £48 : 17 : 2, for the answer required, in a *single* payment. And if this latter sum be divided by 13·297, it will give 3·674, or £3 : 13 : 6, for the value of the same in *annual* payments. Had the age of the man been 56, the value of the assurance in a *single* payment would have been equal to 58·781; which divided by 10·717 would give 5·485 for the value of the same in *annual* payments.

But if the sum had depended on the death of a woman aged 40, its value in a *single* payment would have been equal to 40·765, or £40 : 15 : 4. And this sum, divided by 15·401, would give 2·647, or £2 : 12 : 11, for the value of the same in *annual* payments. And had the age of the woman been 50, the value of the assurance in a *single* payment would have been equal to 49·812; which divided by 13·049 would give 3·817 for the value of the same in *annual* payments.

Example 3. What sum ought to be given for the assurance of £100 on two *joint* lives aged 20 and 40: interest at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on the two joint lives is, by Table IX, equal to 12·545, which being multiplied by ·045 (or the rate of interest) will produce ·56452;

this quantity, subtracted from unity, leaves ·43547, which being divided by 1·045 (or the amount of £1 in a year) will give ·41672; and this multiplied by 100 will produce 41·672, or £41 : 13 : 5, for the answer required, in a *single* payment. If this latter quantity be divided by 13·545, it will give 3·077, or £3 : 1 : 6, for the value of the same in *annual* payments.

Had the two lives been 50 and 70, the value in a *single* payment would have been equal to 71·936; which being divided by 6·517 would give 11·035 for the value of the same in *annual* payments.

Had the assurance been made on the joint lives of a man aged 46, and his wife aged 40, the value of the same (reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden*) would have been equal to 56·592 in a *single* payment: and which, being divided by 11·286, would give 5·014 for the value in *annual* payments.

Or, had these two lives been respectively 56 and 50; the value would in such case have come out equal to 65·869 in a *single* payment: and which being divided by 8·874 would give 7·423 for the value in *annual* payments.

Example 4. What is the value of an assurance of £100 on the *longest* of two lives aged 20 and 40; interest $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of an annuity on the longest of two lives aged 20 and 40 is, by Question VIII, equal to 18·333;

which, being multiplied by ·045 and subtracted from unity, leaves ·17502; this quantity, divided by 1·045, will give ·16748, and which being multiplied by 100 will produce 16·748, or £16 : 15 : 0, for the answer required, in a *single* payment. If this latter quantity be divided by 19·333 (or unity added to the value of an annuity on the *longest* of the two lives) it will give ·866 or 17*s* : 4*d* for the value of the same in *annual* payments, to be made at the beginning of each year during the continuance of either of the given lives.

Had the two lives been 50 and 70 years of age, the value in a *single* payment would have been equal to 41·328; which, being divided by 13·625, would give 3·033 for the value of the same in *annual* payments.

Or, had the assurance been made on the longest of the two lives of a man aged 46 and his wife aged 40, the value of the same (reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden*) would have been equal to 33·031 in a *single* payment: and which, being divided by 17·412, would give 1·897 for the value of the same in *annual* payments.

And had these two lives been respectively 56 and 50, the value would in such case have come out equal to 42·723 in a *single* payment: which being divided by 14·892 will give 2·869 for the value of the same in *annual* payments.

Example 5. What is the present value of a legacy* of £100 to be received on the extinction of any *one*

* I consider a legacy as not due till the *end* of the year in which the testator dies: for it is seldom paid *immediately*.

of three lives aged 20, 30, and 40: reckoning interest at 4 per cent, and the probabilities of living as at *Northampton*?

The value of an annuity on the three joint lives is, by Table XLIV, equal to 8·986, which, being multiplied by ·04, and the product subtracted from unity, leaves ·64056: this quantity divided by 1·04 will give ·61592, and which being multiplied by 100 will produce 61·592, or £61 : 11 : 10, for the present value of the legacy required.

In like manner we might determine the value of the legacy payable on the extinction of any *two* of the three lives above mentioned. For, the value of an annuity on any two out of those three lives is, by Question X, equal to 15·315: consequently, by proceeding as in the last case, we shall find that 37·250, or £37 : 5 : 0, will be the present value of the legacy in this case required.

So also we might find the value of the legacy payable on the extinction of all the *three* lives. For, the value of an annuity on the longest of the lives is, by Question IX, equal to 19·710: and by proceeding in a similar manner it will be found that 20·346, or £20 : 6 : 11, is the present value of the legacy in this case required.

SCHOLIUM.

§ 416. It may be here necessary to advert to the remark which I have made, in the Scholium in page 160, respecting the relative values of a reversionary *sum*, and

a corresponding reversionary *estate:* and which may be verified by a comparison of the values in any two similar cases found by the rules in the two preceding Questions. Thus, by the first example in Question XXVII, it appears that the value of £100 payable on the decease of a person aged 20, interest at $4\frac{1}{2}$ per cent, is equal to 22·234 pounds: and by the first example in Question XXVI, it appears that the value of a corresponding estate (or perpetuity) of £4 : 10 per annum is equal to 25·191 pounds. But the latter is to the former value, in the proportion of 1·045 to 1; and *vice versâ*, the former is to the latter value, as 1 is to 1·045.

QUESTION XXVIII.

§ 417. To find the value of a *Deferred assurance* of any given sum, on any given lives.

SOLUTION FIRST.

For Single and Joint Lives.

Find the value of the assurance of the given sum on the same number of single or joint lives as the given lives, but each older than such lives by the given term; multiply this value by the expectation that the given single or joint lives will receive £1 at the end of that term: the product multiplied by the given sum will be the value required. See the note in page 147.

Example 1. What is the present value of £100

to be received on the death of a man aged 46, provided that should happen *after* 10 years: interest being reckoned at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of an assurance of £100 on a man aged 56 is, by Question XXVII, equal to 58·781; and the expectation that a man aged 46 will receive £1 at the end of 10 years is, by Question II, equal to ·5241; consequently these two quantities being multiplied together will produce 30·807, or £30 : 16 : 2, for the answer required.

Had the assurance been on a woman aged 40, its present value would have been equal to 49·812 multiplied by ·5748; that is, equal to 28·632.

Example 2. What is the present value of £100 to be received on the extinction of *either* of two lives aged 20 and 40, provided that should happen *after* 30 years: interest at $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of the assurance on two joint lives aged 50 and 70 is, by Question XXVII, equal to 71·936; and the expectation of two joint lives, aged 20 and 40, receiving £1 at the end of 30 years is, by Question II, equal to ·0899: therefore the product of these two quantities will give 6·467, or £6 : 9 : 4, for the answer required.

Had the assurance been made on the joint lives of a man aged 46 and his wife aged 40, provided they became extinct *after* 10 years, its present value (taking interest at 4 per cent, and the probabilities of life as

observed in Sweden) would have been equal to 65·869 multiplied by ·4459; that is, equal to 29·371.

SOLUTION SECOND.

For the Longest of any lives.

§ 418. Multiply the value of an annuity on the longest of the given lives, *deferred* for the given term, by the rate of interest; subtract the product from the expectation of the longest of the given lives receiving £1 at the end of the same period, and divide the difference by the amount of £1 in a year: the quotient multiplied by the given sum will be the answer required. See § 182.

Example 3. What is the value of £100 to be received on the extinction of the longest of two lives, aged 20 and 40, provided that shall happen *after* 30 years: interest at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of a deferred annuity for 30 years on the longest of the two lives is, by Question XI, equal to 2·560; which being multiplied by ·045 produces ·1152; and this subtracted from ·2266 (or the expectation that one or other of these two lives will receive £1 at the end of 30 years, as found by Question II) will leave ·1114, which being divided by 1·045 will give ·10660; and this last value, multiplied by 100, will produce 10·660, or £10 : 13 : 2, for the answer required.

Had this sum depended on the extinction of the longest of two lives, a man aged 46 and a woman aged 40, provided that event happened *after* 10 years, the value of the same (reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden*) may be found in a similar manner. For, the value of a deferred annuity for 10 years on the longest of these lives is, by Question XI, equal to 8·508, which being multiplied by ·04 produces ·3403; and this subtracted from ·6530 (or the expectation that one at least of the two lives will receive £1 at the end of 10 years, as found by Question II) will leave ·3127, which being divided by 1·04 will give ·30067; and this last value, multiplied by 100, will produce 30·067, or £30 : 1 : 4, for the answer required.

QUESTION XXIX.

§ 419. To find the value of a *Temporary assurance* of a given sum on any given lives.

SOLUTION.

From the value of the assurance of the given sum on the *whole continuance* of the given lives, subtract the value of a *deferred* assurance of the same sum for the given term: the difference will be the value required. See § 184.

Example 1. What is the present value of £100 to be received on the death of a man aged 46, pro-

vided that shall happen *within* 10 years: interest being reckoned at 4 per cent, and the probabilities of life as observed in *Sweden?*

The value of an assurance of £100 on the whole continuance of this life is, by Question XXVII, equal to 48·858; and the value of a similar assurance, deferred for 10 years is, by Question XXVIII, equal to 30·807, which being subtracted from the former value will leave 18·051, or £18 : 1 : 0, for the answer required.

This is the value in a *single* payment: but if we wish to find the corresponding value in *annual* payments, we must divide this sum by 7·680 (or unity added to the value of an annuity on the given life for one year less than the given term, as found by the rule given in the note, page 380), which will give 2·350, or £2 : 7 : 0, for the value of the same assurance in *annual* payments.

Had the assurance been made on a woman aged 40, its present value would have been equal to the difference between 40·765 and 28·632: that is, 12·133, or £12 : 2 : 8, would have been the answer required, in a *single* payment. And this value divided by 7·900 (or unity added to the value of an annuity on the life for 9 years, as found by the rule above mentioned) will give 1·536, or £1 : 10 : 9, for the value of the same in *annual* payments.

Example 2. What is the value of an assurance of £100 on the *joint* lives of two persons aged 20 and

40, for 30 years: interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of the assurance on the whole continuance of the lives is, by Question XXVIII, equal to 41·672; and the value of a similar assurance deferred for 30 years is, by Question XXVIII, equal to 6·467: therefore this latter value, subtracted from the former, will leave 35·205, or £35 : 4 : 1, for the answer required, in a *single* payment. And this sum divided by 12·959 (or unity added to the value of an annuity on the two joint lives for twenty-nine years) will give 2·717, or £2 : 14 : 4, for the value of the same assurance in *annual* payments.

Had the assurance been made for 10 years on the joint lives of a man aged 46 and his wife aged 40, the value of the same (reckoning interest at 4 per cent, and the probabilities of living as observed in *Sweden*) would have been equal to the difference between 56·592 and 29·371: that is, 27·221 would have been the value in a *single* payment. And this sum divided by 7·329 (or unity added to the value of an annuity on the two joint lives for nine years) will give 3·714 for the value of the *annual* payments.

Example 3. What sum ought to be given for the assurance of £100 for 30 years on the *longest* of two lives, aged 20 and 40: interest at $4\frac{1}{2}$ per cent, and the probabilities of life as observed by *M. De Parcieux?*

The value of the assurance for the whole continuance of the given lives is, by Question XXVII, equal to

16·748; and the value of a similar assurance deferred for 30 years is, by Question XXVIII, equal to 10·660: this latter value therefore subtracted from the former will leave 6·088, or £6 : 1 ; 9, for the answer required, in a *single* payment. And this sum divided by 16·546 (or unity added to the value of an annuity on the longest of the two lives for twenty-nine years) will give ·368, or 7*s* : 4*d*, for the value of the same in *annual* payments.

Had it been required to determine the value of the assurance, in annual payments during the *joint* continuance of the given lives, then 6·086 divided by 12·959 (or unity added to the value of an annuity on the two joint lives for twenty-nine years) would give ·470 for the answer required.

In like manner may be determined the value of an assurance of £100 for 10 years on the longest of two lives, viz a man aged 46 and a woman aged 40; reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden*.

For, the value of such assurance will be equal to the difference between 33·031 and 30·067; that is, 2·964 will be the value in a *single* payment: and this sum divided by 8·251 (or unity added to the value of an annuity on the *longest* of the two lives for nine years) will give ·359 for the value of the same in *annual* payments during the *longest* of the given lives: or being divided by 7·329 (equal to unity added to the value of an annuity on the two *joint* lives for nine years) will give ·404 for the value of the same in

annual payments during the existence of the *joint* lives.

SCHOLIUM.

§ 420. When we have to determine the value of a temporary assurance for a very short term, such as 1, 2, 3, &c years, it will be the easiest method to calculate the value of each year's expectation from the tables of mortality. For, the probability that a person of any given age will die in any particular year is a fraction whose denominator is the number of persons living at that age, and whose numerator is the number of persons that die within the given year; and which fraction, being multiplied by the present value of the given sum due at the end of the given year, will give the expectation of receiving such sum at the end of that year, provided the given life becomes extinct in that year: and the sum of these annual expectations for the 1st, 2nd, 3rd, &c years will be the value of the assurance for those periods respectively.

Example 1. What is the value of an assurance of £100, for *one* year, on the life of a woman aged 40; or, in other words, what is the present value of £100 to be received at the end of the year, provided such life be then extinct: interest being reckoned at 4 per cent, and the probabilities of life as observed in *Sweden*?

The probability that a woman of 40 will die within the *first* year is, by Table XIV, equal to $\frac{65}{4733}$; and the present value of £100 to be received at the end of a year is, by Table LVIII, equal to 96·154: these

two quantities, multiplied together, will produce 1·321, or £1 : 6 : 5, for the value required.

In like manner it may be found that the probability of a woman, aged 40, dying within the *second* year is by the same Table equal to $\frac{78}{4733}$; and that the present value of £100 to be received at the end of two years is equal to 92·456: which quantities being multiplied together will produce 1·465 for the present value of £100 to be received at the end of the second year provided the given life becomes extinct in that year. And this value, added to the one above found, will give 2·786, or £2 : 15 : 9, for the value of the assurance for *two* years.

By a similar method of proceeding it will be found that $\frac{76}{4733}$ multiplied by 88·900 will give 1·428 for the present value of £100 to be received at the end of the *third* year on a similar contingency: and which value, being added to the sum of the two former ones, will give 4·214 for the value of the assurance for *three* years. And so on for the subsequent years.

Had it been required to determine the value of a similar assurance on the life of a man, aged 46, the expectations for the first, second, and third years would have been equal to $\frac{80}{3991}$ multiplied by 96·154, 92·456, and 88·900 respectively:* whence, those expectations would have come out equal to 1·927, 1·853, and

* Because $\frac{80}{3991}$ is not only the probability that such life will die in the *first* year; but also the probability that it will die in the *second* year; and also in the *third* year: as may be seen by Table XIV.

1·782: and the value of the assurance, for one, two, and three years, would have been 1·927, 3·780, and 5·562 respectively.

The same observations will apply to assurances for one year on any *joint* lives. For, the probability that any two joint lives will fail within the first year is the difference between unity and the product of the probabilities that they shall both live to the end of the year: and which difference, being multiplied by the present value of the given sum due at the end of that year, will give the expectation of receiving such sum at that period provided either of the given lives be then extinct.

Example 2. What is the present value of an assurance of £100 for *one* year, on the joint lives of a man aged 46 and his wife aged 40: interest at 4 per cent, and the probabilities of life as observed in *Sweden?*

The probability that a man aged 46 will live to the end of the year is equal to $\frac{3911}{3991}$; and the probability that a woman aged 40 will live to the end of the same period is equal to $\frac{4668}{4733}$: these two fractions, therefore, being multiplied together and their product subtracted from unity, will leave ·0335; which being multiplied by 96·154 (or the present value of £1 due at the end of the year) will produce 3·221, or £3 : 4 : 5, for the value required.

§ 421. These examples will show the method of proceeding in all similar cases: and for the informa-

tion of the reader I shall here subjoin a table of the sums demanded by the different Assurance Companies for the assurance of £100 for *one year* on a single life at the several ages therein mentioned: to which I shall add the fair value that *ought* to be given for the same, according to the probabilities of life as observed by *M. De Parcieux*,* and reckoning interest at 4 per cent.

Ages.	Northampton. 3 per cent.	De Parcieux. 4 per cent.
10	·890	·929
20	1·362	·900
30	1·661	1·037
40	2·030	1·049
50	2·753	1·431
60	3·906	2·983
70	6·184	5·289

From which it appears that the several Assurance Companies require, in most cases, half as much again as ought to be given; and in some cases nearly *double* the sum that should be given for the assurance. And though some compensation ought to be allowed for the expenses incurred in carrying on the business of the Office, as well as a proper remuneration for the services of those who conduct it; yet it is evident that these sums are greater than ought reasonably

* The probabilities here alluded to are, in this particular case, taken from the Table of Observations given by Dr. Price in his *Obs. on Rev. Pay.* vol. ii. p. 456: because the *decrements* of life are there more correctly given; and, being on a more enlarged scale, are therefore more applicable to the present examples.

to be taken: particularly when it is considered that those who insure at *any** of the Offices, for a *term* of years only, have not much prospect of deriving any advantage from the profits of the concern.

QUESTION XXX.

§ 422. To find the value of an *Assurance* of a given sum to be received on the decease of A, provided he dies *before* another given life B.†

SOLUTION.‡

Let O represent a life one year *older* than A; and Y a life one year *younger* than A. Add unity to the value of an annuity on the two joint lives OB, and

* For, such persons do not (even at the Equitable Society) participate in the profits, unless a bonus happens to be declared during the term for which they are assured: which, in most ordinary cases, (if it occurs at all) is but a *partial* advantage.

† The present Question will be found of considerable utility in enabling us to determine the propriety and advantage of those Schemes which are formed with a view of providing *sums* of money to be paid to *Widows*, on the decease of their husbands. See what has been already said on the subject of *annuities*, applied in a similar manner, in the note in page 376.

‡ When the two lives are of the *same* age, the present value required is in all cases equal to the present value of half the sum payable on the extinction of the two joint lives AB. Thus, if the two lives, in the first example, were both aged 20, the present value required would be equal to 17·695: and if they had both been 40 years of age, the required value would have been equal to 22·634.

multiply the sum by the number of persons living at the age of O; then divide this product by the amount of £1 in a year, and reserve the quotient. Multiply the value of an annuity on the two joint lives YB by the number of persons living at the age of Y: and, having subtracted the product from the reserved quotient, divide the remainder by the number of persons living at the age of A. Subtract this last quotient from the present value of £1 payable on the extinction of the two joint lives AB; and the remainder, multiplied by half the given sum, will be the value required. See § 231.

Example 1. What is the present value of £100 payable on the death of A aged 20, provided B aged 40 be then living: interest at $4\frac{1}{2}$ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of an annuity on the two joint lives OB, aged 21 and 40, is, by the rule in page 365, equal to 12·520; which, being added to unity, and then multiplied by 806 (or the number of persons living against the age of 21, in Table III), will produce 10897·120; and this being divided by 1·045 will give 10427·866 for the reserved quotient. The value of an annuity on the two joint lives YB, aged 19 and 40, is, by the rule in page 365, equal to 12·575; which being multiplied by 821 (or the number of persons living at the age of 19, in Table III) will produce 10324·075: this being subtracted from 10427·866 (the reserved

quotient) and the remainder divided by 814 (or the number of persons living at the age of 20) will give ·1275. Now, the value of £1 payable on the extinction of the joint lives AB, aged 20 and 40, is, by Question XXVII, equal to ·4167: therefore ·1275 being subtracted from this value will leave ·2892; which being multiplied by 50 will give 14·46, or £14 : 9 : 2, for the present value required.

Having thus found the present value of the given sum payable on the decease of A, provided B be then alive; we may easily determine the present value of the same sum payable on the decease of B, provided A be then alive. For, we have only in such case to deduct the value, found by the rule, from the value of the same sum payable on the extinction of the joint lives AB. Thus, the present value of £100 payable on the extinction of the joint lives AB is, by Question XXVII, equal to 41·67; whence, if we subtract 14·46 from such value, the difference, or 27·21, will be the present value of £100 payable on the decease of B, provided A be then alive.

These values are, in each case, the sums that ought to be given in a *single* payment: but, if we wish to determine the value of the same in *annual* payments, we must divide those sums by unity added to the value of an annuity on the two joint lives AB. Therefore 14·46 being divided by 13·545 will give 1·068, or £1 : 1 : 4, for the annual payments in the former case: and 27·21 divided by 13·545 will give 2·009, or £2 : 0 : 2, for the annual payments in the latter case.

Example 2. B aged 60 will, if he lives till the decease of A aged 25, be entitled to a legacy of £100: what is the value of his interest in such sum, taking the probabilities of living as at *Northampton*, and the rate of interest 5 per cent?

The value of an annuity on two joint lives OB, aged 26 and 60, is equal to 7·365; which being added to unity and multiplied by 4685 (or the number of persons living at the age of 26, as in Table XXV) will give 39190·025; and this, divided by 1·05, will give 37323·833 for the quotient, to be reserved. The value of an annuity on the two joint lives YB, aged 24 and 60, is equal to 7·399; which being multiplied by 4835 (or the number of persons living against the age of 24) will produce 35774·165: this, being subtracted from the reserved quotient, and the remainder divided by 4760 (or the number of persons living against the age of 25), will give ·32556. Now, the present value of £1 payable on the extinction of two joint lives, aged 25 and 60, is equal to ·60081: therefore ·32556, being subtracted from this value, will leave ·27525: which, being multiplied by 50, will give 13·762, or £13 : 15 : 3, for the present value of B's interest in the legacy.

If this sum be subtracted from 60·081 (or the present value of £100 payable on the extinction of the joint lives AB aged 25 and 60) the difference, or 46·319, will be the present value of the legacy payable on the death of B, provided A be then alive.

And either of these values, divided by unity added

to the value of an annuity on the two joint lives, will show the annual payment which ought to be given by B or A respectively, in order to have the same assured to his heirs provided he dies before the other.

SCHOLIUM.

§ 423. The examples above given show the proper method of proceeding in all similar cases: and it may be here useful to remark that the values in Table LIII (which are adopted by *all* the Assurance Offices in London) have been computed from an incorrect rule given by Mr. Simpson,* and therefore cannot

* In the *Supplement* to his Doctrine of Annuities, Prob. XXXII; and in his *Select Exercises*, Prob. XXXII. In using which rule, it should be observed that, when the reversion is a *sum* and not an *estate*, the value found by the rule must be divided by £1 increased by its interest for a year: as explained in page 162. Agreeably to this correction it will be found that Mr. Simpson's rule may be expressed by the formula $s \times \frac{1-\varrho BB}{2(1+\varrho)} \times \frac{\varepsilon}{e}$: which denotes half the value of an assurance of the given sum payable on the extinction of two joint lives of the same age with the oldest of the two lives, multiplied by a fraction whose numerator is the *expectation of the life* B, and whose denominator is the *expectation of the life* A.

This is the approximate value when B, or the life in expectation, is the oldest of the two lives. But if B be the youngest, this value must be subtracted from $s \times \frac{1-\varrho AB}{(1+\varrho)}$; and the difference will be the value in this case.

I would here observe that Mr. Dodson's formula (*Mat. Rep.* vol. iii. Ques. 23) is deduced from precisely the *same* series as Mr. Simpson's formula: nevertheless, they give *different* results when expounded numerically.

be depended upon when the life of A is very young, or when there is any considerable difference between the ages of the two lives. I shall however enter more at large on this singular fact, in Chapter XIV. See also what has been already said on this subject in the note in page 187.

QUESTION XXXI.

§ 424. To find the value of a *Temporary assurance* of a given sum payable on the decease of A, provided he dies *before* another life B.

SOLUTION.*

Add 2 to the rate of interest; multiply this sum by the value of an annuity on the life B, and add unity to the product. Call this the *first* value.

Add 2 to the rate of interest; multiply this sum by the value of an annuity on a life older than B by the

* It may be necessary to observe that this rule is only an *approximation* to the true value, agreeably to the principles laid down in the note in page 194; and therefore must be always used, not only with caution but, with a due regard to the tables of observation employed. The correct value may be obtained by the help of the formula in Prob. XXVII, cor. 4; but, as that formula could not be conveniently expressed in words at length, I have preferred the one, above alluded to, for the illustration of this part of the subject.

given term, add unity to the product, and then multiply this sum by the expectation of B's receiving £1 at the end of the term. Call this product the *second* value.

Divide the probability that A will die before the end of the term, by the number of years, and multiply the quotient by half the given sum. Call this product the *third* value.

Subtract the *second* value from the *first*, and divide the remainder by the amount of £1 in a year: the quotient thence arising, being multiplied by the *third* value, will give the present value of the given sum required.

Example 1. What is the present value of £100 payable on the decease of A aged 7, provided that shall happen within 14 years, and provided another life B, aged 30, be then alive: interest at 3 per cent, and the probabilities of living as observed at *Northampton?*

The value of an annuity on the life B, aged 30, is, by Table XXVII, equal to 16·922; which being multiplied by 2·03 (or 2 added to ·03) will produce 34·35166; and this, being added to unity, will give 35·35166 for the *first* value.

The value of an annuity on a life 14 years older than B (that is, on a life aged 44) is equal to 13·929; which, being also multiplied by 2·03, will produce 28·27587. This, being added to unity, will give 29·27587; which being multiplied by ·50144 (or the expectation of B's receiving £1 at the end of

14 years*) will produce 14·68009 for the *second* value.

The probability that A will die before the end of the given term is equal to ·14599,† which being divided by 14 will give ·010428; and this quotient, multiplied by 50 (or half the given sum) will produce ·5214 for the *third* value.

The difference between the *first* and *second* value is 20·67157; which, being divided by 1·03, will give 20·0695. This quotient, multiplied by ·5214 will produce 10·464 for the value required.

This is the sum that ought to be given for the assurance in a *single* payment: but, if we wish to determine the value of the same in annual payments, we must divide this sum by 9·566 (or unity added to the value of an annuity on the two joint lives for 13 years); which will give 1·094 for the annual payment required.

Example 2. B aged 60 will, if he lives to the decease of A aged 25, be entitled to a legacy of £100, provided that event shall happen within 15 years: what

* The probability that B shall live to the end of 14 years is, by the rule in Question I, equal to $\frac{3126}{4385}$; and the present value of £1 certain to be received at the end of that period is, by Table LVIII, equal to ·6611: the product of these two quantities will give ·50144 for the expectation required.

† The probability that A shall *live* to the end of 14 years is, by the rule in Question I, equal to $\frac{5060}{5925}$; that is, equal to ·85401. This value, subtracted from unity, will give ·14599 for the probability that A shall *die* before the end of that period.

is the value of his interest therein, reckoning the probabilities of life as observed at *Northampton*, and the rate of interest at 5 per cent?

The value of an annuity on the life aged 60 is, by Table XXVII, equal to 8·392; which, being multiplied by 2·05 and added to unity, will make 18·2036 for the *first* value.

The value of an annuity on a life aged 75 is equal to 4·744; which being multiplied also by 2·05, and added to unity will make 10·7252: and this, being multiplied by ·19637 (or the expectation of B's receiving £1 at the end of 15 years*) will produce 2·1061 for the *second* value.

The probability that A will die before the end of 15 years is equal to ·23634†; which, being divided by 15, will give ·015756; and this quotient, multiplied by half the given sum, will produce ·7878 for the *third* value.

The difference between the *first* and *second* value is 16·0975; which, being divided by 1·05, will give 15·3310. This quotient, multiplied by ·7878, will produce 12·078 for the value required, in a *single* payment.

And this sum being divided by 7·592 (or unity

* The probability that B shall live to the end of 15 years is equal to $\frac{830}{2038}$, and the present value of £1 to be received at the end of that term is equal to ·48102: the product of these two quantities will give ·19637 for the expectation required.

† The probability that A will die in 15 years is equal to $\frac{3635}{4760}$ subtracted from unity; that is, equal to $\frac{1125}{4760}$, or ·23634.

added to the value of an annuity on the two joint lives for 14 years) would give 1·591 for the value of the same sum, if required, in *annual* payments.

SCHOLIUM.

§ 425. If the term, for which the assurance is made, happens to fall within the limits of equal decrements, of the life A, as found in the given table of observations, it is obvious (from the method of deduction in the note, in page 192, &c) that this rule will give the *exact* value. This is the case in the second example here given: for, by referring to Table XXV, it will be found that from the age of 25 to 40 the decrements of life are exactly equal: and consequently the rule is in this case strictly correct. Nevertheless, if the value of the same assurance be found by the help of the formula given in Prob. XXVII, cor. 4, it will come out equal to 12·139: and I can account for this difference in no other way, than by supposing there is some error in the tables of the values of the annuities; for, it is evident that the two results *ought* to be precisely the same.

QUESTION XXXII.

§ 426. To find the value of an *Assurance* of a given sum payable on the decease of A, provided he dies *after* another life B.

SOLUTION.

From the value of the assurance of the given sum payable on the decease of A, subtract the value of the same assurance payable on the decease of A, provided he dies *before* B: the difference will be the value required. See § 241.

Example 1. What is the present value of £100 payable on the decease of A aged 20, provided B aged 40 be then dead: reckoning interest at 4½ per cent, and the probabilities of living as observed by *M. De Parcieux?*

The value of the assurance of £100, payable on the decease of A, is, by Question XXVII, equal to 24·107; and the value of the same sum, payable on the decease of A provided he dies before B, is, by Question XXX, equal to 14·460: consequently 9·647, or £9 : 12 : 11, will be the value required.

Having thus found the value of the given sum payable on the decease of A, provided B be then dead, we may easily determine the value of a similar sum payable on the decease of B, provided A be the extinct: for, we have only in such case to deduct the value, found by the rule, from the value of the same sum payable on the extinction of the longest of the two lives. Thus, the present value of £100 payable on the extinction of the longest of two lives aged 20 and 40 (at the rate of interest &c, above mentioned) is, by Question XXVII, equal to 16·748; from which

if we subtract 9·647, as above found, the difference, or 7·101, will be the value of £100 payable on the decease of B, provided he dies after A.

These sums are the values which ought in each case to be given in a *single* payment: but, if we wish to determine the value of the same in *annual* payments till the claim *is determined*, we must divide the single payment, thus found, by unity added to the value of an annuity on the two joint lives. Or, if we wish to determine the value of the same in *annual* payments till the claim *becomes due*, we must divide the single payment by unity added to the value of an annuity on the single life, on which the assurance is made.* Thus, 9·647 being the value, in a *single* payment, of an assurance of £100 on the life A provided he dies after B; it follows that the value of the same assurance, in *annual* payments till the claim *is determined*, is equal to 9·647 divided by 13·545: that is, equal to ·712, or 14*s* : 3*d*. And that the value of the same assurance, in *annual* payments till the claim *becomes due*, is equal to 9·647 divided by 17·624: that is, equal to ·547, or 10*s* : 11*d*.

Example 2. What is the present value of £100 payable on the decease of A aged 25, provided he should die after B aged 60: interest at 5 per cent, and the probabilities of living as at *Northampton*?

The value of an assurance of £100 on the decease of A is, by the rule in Question XXVII, equal to

* See page 353.

30·633; and the value of the same sum payable at the same period, provided B be then extinct, is, by Question XXX, equal to 13·762: consequently 16·871, or £16 : 17 : 5, will be the value required, in a *single* payment.

This value, being divided by 8·383 will give 2·013, or £2 : 0 : 3, for the *annual* payments till the claim *is determined:* or, being divided by 14·567, will give 1·158, or £1 : 3 : 2, for the *annual* payments till the claim *becomes due.*

QUESTION XXXIII.

§ 427. To find the value of a *Temporary assurance* of a given sum payable on the decease of A, provided he dies *after* another life B.

SOLUTION.

From the value of a temporary assurance of the given sum payable on the decease of A, subtract the value of a similar temporary assurance, payable on the decease of A provided he dies *before* B: the difference will be the value required.

Example. What is the present value of a legacy of £100 payable on the decease of A aged 25, provided he dies within 15 years, and provided also that another person B, now aged 60, be then dead: in-

terest being reckoned at 5 per cent, and the probabilities of living as at *Northampton?*

The value of an assurance of £100 payable on the decease of a person aged 25, provided that should happen within 15 years, is, by the rule in Question XXIX, equal to 16·354*; and the value of a similar assurance, provided another person now aged 60 be then living, is, by Question XXXI, equal to 12·078: consequently this latter value subtracted from the former will leave 4·276, or £4 : 5 : 6, for the value of the assurance required.

SCHOLIUM.

§ 428. This rule will still be correct although the given term *exceed* the number of years between the age of B, and the oldest life in the table of observations: but, in such case, the value of the assurance payable on the decease of A provided he dies *before* B (being now for the *whole continuance* of the joint lives) must be found by Question XXX instead of Question XXXI: and the value thus found, being deducted from the temporary assurance on the life A, will give the value required†.

* The value of the assurance on the *whole continuance* of the life is, by the rule in Question XXVII, equal to 30·633; and the value of the same assurance *deferred* for 15 years is, by the rule in Question XXVIII, equal to 14·279: consequently the difference between these two values, will be the value of the *temporary* assurance.

† Mr. Morgan has given a singular and troublesome rule for this case, in Dr. Price's *Obs. on Rev. Pay.* vol. 1, page 70, note i. It is neither simple nor *correct.*

Thus, if the term in the preceding example had been 40 years; then the value of the assurance on the life A for 40 years would, by the rule in Question XXIX, be equal to 27·682*; and the value of the assurance on the same life, provided B be alive at his decease, would, by Question XXX, be equal to 13·762; consequently, this latter value subtracted from the former would leave 13·920, or £13 : 18 : 5, for the value required, in a *single* payment. And 13·920 divided by 8·383 (or unity added to the value of an annuity on the two joint lives) will give 1·661 for the value of the same in *annual* payments till the *claim* is determined: or, being divided by 14·164 (or unity added to the value of an annuity on the life A for 39 years) will give ·983 for the same value in *annual* payments till the sum *becomes due*.

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§ 429. Before I close the present chapter I shall insert the solution of a Question, which will be often found of considerable practical utility, not only to individuals, but likewise to those Societies whose business consist in granting Assurances on lives.

The reader must be already aware that if a person were to make an assurance at any of the Offices on his

* The value of the assurance on the *whole continuance* of the life is, as in the preceding note, equal to 30·633; and the value of the same assurance *deferred* for 40 years is, by the rule in Question XXVIII, equal to 2·951: consequently the latter value subtracted from the former will be the value of the *temporary* assurance required.

own life for a *single* year, and to repeat this at the end of every successive year to the utmost extremity of life, the annual payment (for such assurance) would be continually *increasing* till his death. But, if he made the assurance on the *whole continuance* of his life, and contracted with the Office to pay the value of such assurance by *equal* annual payments during his life (as is usually the case), it is evident that such annual payment ought to be *greater* than the premium required for an assurance for a single year at his *present age*, but *less* than the premium required for a similar assurance at the more *advanced periods* of life. Hence, it appears that if a person, who was originally assured for the whole term of his life, should be desirous (either through inability, or any other motive) of renouncing his claim upon the Office and of cancelling his policy, he ought to have some part of those annual payments returned to him: or, in other words, a compensation ought to be made him for that *excess* in the annual payments which he has been advancing to the Society. The object of the following question is to determine the amount of that remuneration.

QUESTION XXXIV.

§ 430. To find the sum that ought to be given to a person, who is assured for the whole term of his life, for a given sum, in order that he may *renounce his claim* thereto.

SOLUTION.

Multiply the equal annual payment, which he has been giving since the assurance commenced, by the value of an annuity (increased by unity*) on the life at its present age; subtract the product from the value of the assurance of the given sum on the life at its present age: the difference will be the sum required.

Example. A person *now* aged 50, who has been paying 21·790, or £21 : 15 : 10,† annually for the assurance of £1000 at his death, is desirous of discontinuing the same; what sum ought to be given to him, by the Office, as a compensation for so doing: interest being reckoned at 3 per cent, and the probabilities of living as at *Northampton?* ‡

The value of an annuity (increased by unity) on a life aged 50 is, by Table XXVII, equal to 13·436; which being multiplied by 21·790, will produce 292·770: this being subtracted from 608·660 (or the

* This supposes that the policy is cancelled immediately *before* the annual payment becomes due: but if immediately *after*, we must multiply the payment, above alluded to, by the value of an annuity on the given life, without the addition of unity.

† This is the annual payment for the assurance of £1000 on a life aged 20, as appears by Table LI.

‡ The rate of interest and probabilities of life, in *such* computations, *ought* to be the *same* as those adopted by the Office at which the policy is effected.

value of the assurance on a life aged 50, as appears by Table LI) will leave 315·890, or £315 : 17 : 10 for the answer required.*

SCHOLIUM.

§ 431. If the sum, which is to be received at the death of the assured, has been increased by any addition, or bonus, (as will oftentimes be the case in the *Equitable Society*) we must subtract the product above alluded to, from the value of the assurance of the given sum *together with its bonus;* and the remainder will, in this case, be the sum required. †

Example. Suppose that the several additions made to the policy, alluded to in the preceding example,

* The truth of this rule will be evident from the following statement. The Society may be considered as indebted to the assured in the present value of an assurance of £1000 on a life aged 50; which is equal to 608·66, or £608 : 13 : 3. And the assured may be considered as owing to the Society the present value of all the annual payments of £21 : 15 : 10, during the remainder of his life; the first of which payments is supposed to be made immediately: therefore the value of all those payments will be equal to 21·790 multiplied by 13·436; which produces 292·77, or £292 : 15 : 5. Consequently the interest of the assured in his policy will be equal to the *difference* between £608 : 13 : 3, and £292 : 15 : 5 : that is, equal to £315 : 17 : 10, as found by the example in the text.

† Or, we may add the value of an assurance of the *additional* sum on the given life, to the value found by the preceding solution; which will give the same results.

amount to £1800; * in which case, the executors of the assured would be entitled to £2800 on his death: what sum ought now to be given him for renouncing his claim upon the Society; the interest, &c, being as in the preceding example?

The value of an assurance of £2800 on a life aged 50 is 1704·248; from which we must subtract 292·770: the difference is 1411·478, or £1411: 9:7, which is the answer required.

The same result would have been obtained by adding £1095: 11: 9 (or the value of an assurance of the additional £1800 on the given life) to £315: 17: 10, as found by the preceding example.

§ 432. These two examples will show the method of proceeding in all similar cases, whether the assurance depends upon a single life, upon any joint lives, or on any other contingency. It will also serve to show the amount of the *debts* owing by a Company, whose business consists in making Assurances on lives: since these are the sums which would be required to cancel the respective claims on the Society, and may consequently be fairly considered as money owing by them. These debts, therefore, being deducted from the amount of capital in hand, will leave the net *surplus* stock belonging to the Society: and it is this net sur-

* This would actually be the case at the *Equitable Society*. So that this person, although originally assured there for *one thousand* pounds only, would now be entitled to receive above *one thousand four hundred* pounds for *cancelling his policy!!!*

s stock *alone* that can be considered as the true *fits* of the concern, and as the *only* fund from which ' divisions ought to be made amongst the different mbers; either by way of interest, dividend, or ıus. A society that is not guided by some principle this kind must inevitably terminate in disgrace l ruin.

CHAPTER XIII.

On Schemes for providing Annuities for the Benefit of Old Age, and of Widows.

1. *For Old Age.*

§ 433. THE rule given for the solution of Question VI, in the preceding Chapter, is of considerable practical use; since, by means of it, we are enabled to determine the efficacy and propriety of those schemes and establishments which are proposed with a view of providing *Annuities for Old Age.* For, having found the present value of an annuity of £1 per annum on any given life or lives, to commence at the end of any number of years from the present time, we may easily find the present value of any *other* annuity (either in a *single* payment, or in *annual* payments, or in *both*) by multiplying the present values, thus deduced, by that other annuity; agreeably to the principles already laid down in the note in page 362. And in like manner, having found the present value above alluded to, we may readily determine the annuity that ought to be given, at the end of any period, for a given sum paid down immediately; or, for a given sum, *part* of which is paid immediately, and the *remainder* by annual payments till the end of the given term.

Thus, it appears, by the second example in page 378, that the present value of an annuity of *one pound* per annum on the life of a female aged 40, to commence at the end of 10 years, is (reckoning interest at 4 per cent, and the probabilities of life as observed in *Sweden*) equal to 6·926, or £6 : 18 : 6: consequently an annuity of £44 per annum to commence at the same period would be equal to 304·744, or £304 : 14 : 11, in a *single present* payment. And this latter sum divided by 7·900 (or unity added to the value of an annuity on the given life for 9 years) will give 38·575, or £38 : 11 : 6, for the value of the same annuity in *annual* payments: the first of such payments to be made immediately, and the remaining ones at the beginning of every subsequent year, provided the given life is then in being. Or, if £200 (part of the £304 : 14 : 11) be paid down *immediately*; then the *annual* payments which ought in this case to be made, in order to supply the deficiency, will be equal to the remaining sum of 104·744 divided by 7·900:* that is, equal to 13·259, or £13 : 5 : 2.

* If the first of these annual payments is not made till the *end* of the first year, then the sum here alluded to must be divided by 7·475 (or the value of an annuity on the given life for 10 years) which will make the annual payments equal to 14·013, or £14 : 0 : 3: and, in this case, the last annual payment will be made at the same time that the first payment of the annuity becomes due. In the cases mentioned by Dr. Price (*Obs. on Rev. Pay.* vol. i. p. 141) and to which I shall presently allude, the payments were made *half* yearly; and the first of those half-yearly payments was paid down with the Admission money: but he does not notice this fact, although it makes some difference in the results.

In like manner if we wish to determine the *annuity* that ought to be granted on such life at the end of 10 years, for £73 : 10 : 0, paid down *immediately*, and £6 : 14 : 0 in *annual* payments, we must proceed as follows. Multiply 6·700 (or the amount of the annual payment) by 7·900 (or unity added to the value of annuity on the given life for 9 years)*, which will produce 52·930, or £52 : 18 : 7, for the total present value of such annual payments; and which being added to the admission money, or £73 : 10 : 0, will make the whole value, given for the annuity, equal to 126·430, or £126 : 8 : 7. Therefore this quantity, being divided by 6·926 (agreeably to the principles laid down in the note in page 363), will give 18·254, or £18 : 5 : 0, for the amount of the annuity required.

§ 434. This example will show the method of proceeding in all similar cases; and I have taken this one in particular, because it is the one assumed by Dr. Price (*Obs. on Rev. Pay.* vol. i. p. 137) to expose the futility and iniquity of those Schemes that were published by the several societies instituted about the year 1770: who, by holding out a false lure to the public, took in the unwary, and entailed misery and distress on the unfortunate adventurers.† Happily his

* If the first annual payment is not to be made till the *end* of the year, we must multiply 6·700 by 7·475 (or the value of an annuity on the given life for 10 years): see the preceding note.

† The following are the terms upon which some of those So-

efforts were crowned with success, and none of those public societies now exist to disgrace the present age. But although he has sufficiently demonstrated that those bubbles could not possibly comply with the terms

cieties granted annuities to members, aged 40, for what might happen to remain of their lives after 50 years of age.

	Admission Money. £ s.	*Annual Payment.* £ s.	*Annuity granted.* £ s.
The Laudable Society	73 : 10	6 : 14	44 : 0
The London Annuitants	25 : 0	10 : 0	44 : 0
The Equitable Society of Ann^{s}. .	32 : 0	13 : 0	44 : 0
The London Union Society	37 : 0	7 : 0	54 : 12
The Amicable Society of Ann^{s}. .	28 : 16	6 : 0	26 : 0
The Provident Society	31 : 10	8 : 8	25 : 0

The annual payment was usually paid *half* yearly; and the *first* half yearly payment was made at the time of admission. It will be readily seen (from the example given in the text) that there was no probability that the Societies would ever be able to *continue* these enormous annuities to *all* the members. The truth is, that (even at that day) they were styled *impositions on the public proceeding from ignorance, and supported by credulity and folly.* "But (as Dr. Price justly continues) this is too gentle a censure. "There is reason to believe that *worse* principles have contributed "to their rise and support. The present members, consisting "chiefly of persons in the more advanced ages who have been ad- "mitted on the easiest terms, believe that the schemes they are "supporting will last their time and that they will be gainers. "And, as to the injury that may be done to their successors or to "younger members, it is at a distance, and they care little about it. "Agreeably to this principle the founders of these societies begin "so low as not to require perhaps a *fourth* or a *fifth* of the values "of the annuities they promise. Afterwards they advance gra- "dually, just as if they imagined that the value of the annuities

held out to the public, yet he has not shown that they were *so* deficient and absurd as they will appear to be, when compared with the values deduced as above from the real probabilities of life.* The present examples will be sufficient to enable any one to examine the accuracy and sufficiency of similar proposals: and I shall not detain the reader with any further comments on this subject, as it is probable that a similar circumstance will not soon again occur to call forth the censure of every honest member of society. The public are now better acquainted with the method of calculating the values that ought to be given in such cases, and are not likely to become again in *this* manner the dupes of any artful impostor. Their danger now seems to lie in the *opposite* extreme.

" was nothing determinate, but increased with every increase of " the society. But, as no ignorance can believe this, the true de- " sign appears to be, to form soon as large a society as possible by " leading the unwary to endeavour to be *foremost* in their applica- " tions, lest the advantage of getting in on the easiest terms should " be lost. It is well known that these arts have succeeded won- " derfully; and that, in consequence of them, these societies now " consist of persons who for the *same* annuities make higher or " lower payments, according to the time when they have been " admitted; and the generality of whom, therefore, must know " that either more than the values have been required of the mem- " bers last admitted; or, if not, that they are themselves expecting " considerable annuities for which they have given no valuable " consideration: and which, if paid, must be *stolen* from the " pockets of some of their fellow-members."

* Dr. Price formed his calculations from the values of annuities deduced from the hypothesis of *M. De Moivre.*

I do not, however, by these observations at all intend to discourage institutions or schemes for providing annuities for old age. On the contrary, I am fully persuaded that a society or office, that would proceed upon an efficient and *liberal* plan, might be of essential advantage to the state and highly beneficial to the public. Many persons, particularly in the inferior stations of life, would in such case be induced to lay by (during the period of youth and vigour) many small sums, which are now squandered in riot and dissipation; and by endeavouring to get a little money beforehand would acquire habits of industry, and be probably enabled thus to make provision for themselves in the more advanced periods of life, when they will be incapable of labour. Thereby rendering themselves not only more independent and valuable members of society, but also obviating the necessity of their applying to the parish for relief. Dr. Price justly observes that "the lower orders of mankind are objects "of particular compassion, when rendered incapable, "by accident, sickness, or age, of earning their sub-"sistence. This has given rise to many very useful "societies among them for granting relief to one an-"other, out of little funds supplied by weekly contri-"butions." It is much to be feared, however, that many of these establishments are formed on such vague and inefficient plans, without any regard to the true principles of calculation, that they are not always entitled to our unqualified approbation. When thus erroneously founded, and perhaps at the same time

badly and carelessly conducted, they only serve to increase the misery which they were intended to remove.

§ 435. Under these circumstances it is to be regretted that the Legislature has not adopted some efficient measures towards assisting such persons, in the humbler stations of life, as might be desirous of employing their money in this laudable manner. Two attempts have been made to obtain an Act for this purpose: but, though it has each time passed the House of Commons, it has been as regularly thrown out by the House of Lords. I cannot here, however, omit the opportunity of inserting the pertinent remarks, on this subject, of a very able writer*, which will serve to show the objects that those bills had in view.

'To make such a provision for one's old age, is so 'natural a piece of prudence, that it seems wonderful 'at first sight that it should not be generally practised 'by the labouring poor above described, as it is almost 'universally by persons engaged in the higher paths 'of industry, such as the several branches of com-

* Mr. Baron Maseres, in a pamphlet which he published, entitled *A Proposal for establishing Life Annuities in Parishes for the Benefit of the Industrious Poor:* London, 1772. The learned author has lately favoured me with a copy of that pamphlet, with an obliging offer that I might make what use of it I pleased in my intended publication: and I lament that the limits of the present treatise will not enable me to enter upon the subject more than by an insertion of the above extract from that valuable work.

'merce, and the practice of the liberal professions: 'nor can their negligence in this respect be accounted 'for in any other way so naturally as by ascribing it to 'their wanting opportunities of employing the money, 'they might save, in some safe and easy method that 'would procure them a suitable advantage from it in 'the latter periods of their lives. They know, for 'the most part, but little of the public funds; and 'when it happens that they are acquainted with them, 'the smallness of the sums they would be intitled to 'receive (as the interest of the money they could afford 'to lay out in them) is no encouragement to dispose 'of it in that way. What inducement, for instance, 'can it be to a man, who has saved ten pounds out of 'his year's wages, to invest it in the purchase of three 'per cent Bank annuities, to consider that it will pro-'duce him about six or seven shillings a year? It is 'but the wages of three days labour. And if they 'lend their money to tradesmen of their acquaintance, 'as they sometimes do, it happens not unfrequently 'that their creditor becomes a bankrupt; and the 'money, they had trusted him with, is lost for ever; 'which discourages others of them from saving their 'money at all, and makes them resolve to spend it in 'the enjoyment of present pleasure. But, if they saw 'an easy method of employing the money they could 'spare, in such a manner as would procure them a 'considerable income in return for it at some future 'period of their lives, without any such hazard of 'losing it by another man's folly or misfortune, it is

'probable they would frequently embrace it: and thus 'a diminution of the poor's rate on the estates of the 'rich, an increase of present industry and sobriety in 'the poor, and a more independent and comfortable 'support of them in their old age than they can other-'wise expect, would be the happy consequences of 'such an establishment. Now this, I conceive, might 'be effected in the following plain and easy method.

'1° Let the churchwardens and overseers of every 'parish be impowered by Act of Parliament to grant 'life-annuities to such of the inhabitants of the parish 'as shall be inclined to purchase them, to commence 'at the end of one, two, or three years (or such other 'future period of time as the purchaser shall choose) 'and to be paid out of the poor's rate of the parish, 'so that the lands and other property in the parish 'that is chargeable to the poor's rate, shall be answer-'able for the payment of these annuities. This cir-'cumstance would give these annuities great credit 'with the poor inhabitants, by setting before them so 'solid and ample a security for the regular payment 'of them.

'2° Let the annuities, thus granted to the poor in-'habitants, be such as arise from a supposition that 'the interest of money is three per cent; or some higher 'rate of interest, if the churchwardens or overseers of 'the poor shall think fit to make use of such higher 'interest.

'3° But, at the rate of three per cent, the purchaser 'should have a right to an annuity, and the church-

'wardens and overseers of the poor should be com-
'pellable to grant it.

'4° No annuity, depending upon one life, should
'exceed 20l. a year.

'5° No less sum than *5l.* should be allowed to be
'employed in the purchase of an annuity. This is to
'avoid intricacy and multiplicity of accounts.

'6° An exact register of these grants should be
'kept by the churchwardens and overseers of the poor
'in proper books for the purpose, in which the grants
'should be copied exactly, and the copy of each grant
'subscribed by the person to whom it is granted: or,
'if he cannot write, marked with his mark, and sub-
'scribed by two other persons as witnesses. And this
'copy in the register-book of the parish should be
'good evidence of the purchaser's right to the annuity,
'in case the original deed of grant to the purchaser
'(which was delivered to him at the time of the pur-
'chase) should afterwards be lost.

'7° The money, thus paid to the churchwardens and
'overseers of the poor for the purchase of life-annuities,
'should be employed in the purchase of three per
'cent Bank annuities, in the joint names of all the
'churchwardens and overseers; and by them transferred
'at the expiration of their offices to their successors,
'and so on to the next successors for ever, so as to
'be always the legal property of the churchwardens
'and overseers of the poor for the time being, in trust
'for the persons who should be entitled to the several
'life-annuities, granted in the manner above men-

'tioned; and the interest of this money should be 'received every half year, and invested in the purchase 'of more principal annually, so as to make a perpetual 'fund for the payment of these life-annuities. And 'when any annuity became due, the churchwardens 'and overseers of the poor should pay it out of this 'fund, and should sell a sufficient part of the principal 'for that purpose, where the interest was not sufficient 'for the purpose, as would generally be the case: 'and the deficiencies (if any were) of both principal 'and interest should be made good out of the poor's 'rate assessed upon the parish. But this could hardly 'ever happen, if the annuities granted to the pur-'chasers were such as would be proportional to the 'money contributed, upon a supposition that the in-'terest of money was only three per cent; because that 'is a lower interest than that which the parish would 'receive from the government for the same money, 'by investing it in the three per cent Bank annuities, 'as that stock is now twelve per cent under par,* and 'is not likely soon to rise to par again. So that the 'landholders of the parish, and the owners of other 'rateable property in it, need be under very little ap-'prehension of having their estates exorbitantly bur-'thened by a great increase of the poor's rate, in order 'to make good the payment of these annuities. On 'the contrary, they would be gainers by this institution, 'as was observed above, since many of the poor who

* It is now (1810) above 30 per cent under par.

'must otherwise, in their old age, come to be a bur-
'then upon the parish, would now be maintained, in
'part at least, by annuities paid to them out of a fund
'of their own raising.'

Agreeably to this plan, a bill* was brought into the House of Commons in the year 1773 by Mr. William Dowdeswell, and passed, on a division after a debate, by a majority of two to one of all the members present; but it was rejected by the House of Lords in consequence of the opposition of Lord Cambden, who conceived that the measure might be ultimately injurious to the *landed interest;* since the value of future leases might be affected by an increase of the poor's rate to make good any deficiency arising from the failure or defection of the scheme. A bill of a similar nature, however, with tables computed by Dr. Price,† was introduced in the year 1789; but shared a similar fate,

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2. *For Widows.*

§ 436. The rules, by which Question XIII and XVIII, in the preceding Chapter, are solved, are very useful in enabling us to determine the advantage and propriety of those schemes which are instituted with a

* A copy of this bill, together with the tables that were computed for it, are inserted by Mr. Baron Maseres in his *Doctrine of Life Annuities.*

† These Tables are inserted in Dr. Price's *Obs. on Rev. Pay.* vol. ii. page 473.

view to provide *Annuities for Widows**. For, having found the present value of a reversionary annuity, or a deferred reversionary annuity, of £1 per annum on any given life after any other given life, we may readily find the present value of any other annuity (either in a *single* payment, or in *annual* payments, or in *both*) by multiplying the present value, thus obtained, by that other annuity. And in like manner, having found the present value above alluded to, we may easily determine the annuity which ought to be granted, to the life in reversion, for a given sum paid down immediately; or, for a given sum, *part* of which is paid down immediately, and the *remainder* by annual payments during the existence of the life in possession. Or, according to any other plan which may be proposed.

Thus, let the scheme of a society, for granting annuities to widows, be such that, if a member lives *one* year after admission, his widow shall be entitled to a life annuity of £20: if *seven* years, to £10 more (or £30 in the whole): if *fifteen* years, to £10 more (or £40 in the whole). What *ought* to be the annual payments for members aged 30, 40, and 50 years respectively, and supposing them of the same ages as their wives: interest being reckoned at 4 per cent,

* These observations relate to the method of determining the best mode of providing *Annuities* for Widows: but those inquiries which relate to the best mode of providing for the payment of any given *Sum* to a widow, on the death of her husband, are answered by Question XXX, in the preceding Chapter.

and the probabilities of living as observed in *Sweden* amongst males and females respectively; and also as observed at *Northampton* and at *London?*

By proceeding according to the rules laid down in the Scholium in page 394*, we shall find that the annual payments which ought to be made by the members, of the respective ages above mentioned, will (according to the several tables of observations made use of) be as follow:

Ages.		*Sweden.*	*Northampton.*	*London.*
30	=	6·90	7·66	8·52
40	=	7·89	8·09	9·06
50	=	8·50	8·40	9·51

I have selected this scheme as being that on which

* For example: the value of a reversionary annuity on the life of a female aged 30 after the life of a man aged also 30, and deferred for *one* year provided *both* the lives continue so long, is (by the rule in the Scholium, according to the *Sweden* observations) equal to 3·108: consequently the present value of an annuity of £20, under the same circumstances, is 62·160. In like manner, the present value of £10 per annum on the same life, deferred for *seven* years provided *both* the lives continue so long, is equal to 21·680. And by the same rule the present value of £10 per annum, deferred for *fifteen* years, is under the like circumstances equal to 12·566.

These values, being added together, are equal to 96·404; or the value of the expectation, described in the scheme, in a *single* present payment: and which being divided by 13·965 (or unity added to the value of an annuity on the two joint lives) will give 6·903 for the value of the same expectation in *annual* payments, as stated in the text. In like manner may be found the answer to all Questions of a similar kind.

the *London Annuity Society for the Benefit of Widows* set out in 1765. The *Laudable Society*, which was formed on nearly a similar plan, had been established in the year 1761. In each of these, the *annual* contribution of every member was five guineas only, payable half-yearly; for which payment, his widow was entitled to the annuities stated in the scheme above mentioned, according to the conditions there expressed. Nothing, however, can more fully show the inadequacy of the means for carrying into effect the intentions of those Societies than the examples above given. For it will be seen that, on the supposition that the members are of the *same ages* as their wives, the Societies received, on an average, little more than *three fifths* of the true value of the annuities: but, on the supposition that they are, one with another, *ten years older* than their wives, it will be found that they received only *one half* of the true value of such annuities. The consequences of such inequitable measures were highly injurious. The more early annuitants enjoyed the *full* sums, according to the conditions of the plan: but, after pursuing this pernicious course for some years, the Directors at length listened to the voice of reason; and, in consequence of the repeated warnings that were given them, found it necessary to adopt one of the two following plans in order to maintain the permanency and security of the establishment:—to *increase* the premiums, or *diminish* the annuity. The *London Annuity Society*, at an early period, adopted the former proposal, and thus preserved

its honour and its credit: but the *Laudable Society*, although it repeatedly reduced the annuities, had been too long struggling with the errors of its original establishment to enable it to derive much permanent advantage from this procrastinated remedy. "If circumstances therefore should still continue unfavourable, "the next measure must be the dissolution of the "Society, and a division of the remaining capital "among the annuitants and surviving members, in "proportion to their respective interests in the funds "of the Society*."

§ 437. Such will be the final issue of every scheme that is not founded on correct observations and on mathematical principles: and though my remarks have been confined to the two establishments above mentioned, they will, I fear, apply with too much justice to several others of a similar nature. "There are in "this kingdom many institutions for the benefit of "widows besides the two on which I have now re-"marked; and in general, as far as I have had any "information respecting them, they are founded on "plans equally inadequate, having been formed just "as fancy has dictated, without any knowledge of "the principles on which the values of reversionary "annuities ought to be calculated. The motives, "which influence the contrivers of these institutions,

* See the history of these two Societies, brought down to the present period, in Dr. Price's *Obs. on Rev. Pay.* vol. i. pages 72—104.

" may be *laudable*; but they *ought* (I think) to have " informed themselves better."—" The longer such " schemes are carried on, the more mischief they " must produce. Tis vain to form such establish- " ments with the expectation of seeing their fate de- " termined *soon* by experience. If not more extrava- " gant than any ignorance can well make them, they " *will* go on prosperously for 20 or 30 years; and if " at all tolerable, they *may* support themselves for 40 " or 50 years, and at last end in distress and ruin. " All inadequate schemes lay the foundation of *present* " relief on *future* calamity; and afford assistance to a " *few* by disappointing and distressing *multitudes*.*"

The very learned and able writer, from whose work these quotations are made, employed his great abilities in detecting the pernicious tendency and iniquity of the several schemes above alluded to. His remarks on this subject are invaluable, and will always be consulted with advantage. He has not only shown how far the various societies, then in existence, erred from the true line of equity and propriety (predicting, therefrom, their incapacity and ruin); but has likewise pointed out some of the *best* schemes for providing annuities for widows—such as might be " durable, and " at the same time easy and encouraging." As I cannot add much to the observations of so intelligent an author I must refer the reader to that work, for any further information on this subject.

* Dr. Price's *Obs. on Rev. Pay.* vol. i. Chapter II.

CHAPTER XIV.

An Account of the several Companies, established in London, for Life Assurances, &c.

The Amicable Society.

§ 438. THE first establishment of this kind, in point of time, is the *Amicable Society* for a perpetual Assurance office; which was founded by a charter of Queen Anne in 1706, at a period when the subject of Life Assurances was but little understood. Its original plan was consequently in many instances exceedingly defective, absurd, and inequitable: and, though it has since undergone several partial reforms, it is still liable to many objections. It was limited by its first charter to the disposal of shares or numbers, not exceeding 2000, held by *single* lives; and entitling to claims when they drop. For each of these shares every new purchaser paid down at entrance £7 : 10 : 0, besides £1 : 11 : 0, as the first quarterly payment of £6 : 4 : 0 per annum to be continued during his life. An annual dividend of £1 : 4 : 0 for each share was allowed to every purchaser, out of the profits of the society; which reduced the annual payment, on each share, to £5. No

one person was allowed to purchase more than *three* shares; nor were any admitted to be purchasers whose ages were below 12 or above 45: and all between those ages were admitted on the *same* terms. Every person admitted became a member: and the net annual income, *arising from all the annual payments**, was equally divided among the nominees of such members as happened to die within the year. In 1757 the society engaged that this dividend, though it might be more, should not be *less* on each share than £125; and in 1770 they guarantied that it should not be less than £150.

Such are the outlines of the plan upon which the business of this society was *formerly* conducted. Its absurdity and injustice are manifest in its two most prominent features: 1° in regulating the dividends,

* Making £10,000 when the society was full; being £5 net income on each of the 2000 shares. The *admittance money* of £7 : 10 : 0, together with such profit, interest or improvement, as the Society might be able to make, were carried to a *separate* account. In 1757 these savings had accumulated to £25,300 three per cent stock; which in 1770 had increased to £33,300. Out of this stock, the sum of £4000 Consols and £11,000 Reduced was in 1770 appropriated and set apart, together with all future savings, as a fund for augmenting the claims to the full sum of £150, whenever they fell short of that amount under the dividend directed by the charter. In 1780 the stock of the Society had been increased to £51,300; and in 1790, to £64,300 three per cent annuities, together with some property in houses. Out of this amount, the sum of £30,000 stock was appropriated in aid of the fund above mentioned: but the remainder cannot be disposed of without the consent of two successive General Courts.

among the *nominees*, by the number of members that die within the year; whereby that benefit is made to depend not on the value of the contribution, but on a *contingency:* 2° in requiring, from all the members, the *same* payments without regarding their differences of age. The utility of this institution however (defective as it is) has been virtually acknowledged, and the plan thereof sanctioned, by the grant of three subsequent charters: one in 1730, providing for the better regulation of the society; the next in 1790, enabling it to enlarge the number of its members to 4000;* and the last in 1807, whereby the society is enabled to render itself more extensively useful, and whereby it has obtained several other powers for the better regulation and conduct of the business of the corporation.

By this new charter the society is enabled to grant *twelve* shares on each life at any age between 8 and 67. It is also empowered to proportion the premiums on

* When the Society, in 1770, guarantied that the dividend on each share should not be less than £150, the number of subscribers increased very considerably: the natural consequence of which was, (in a Society so constituted) that, for the first 10 or 15 years, the dividends rose from £150 to £200; and in some years even to £300. A similar rise was produced from the increase of the number of shares, by the charter of 1790: but these effects were *temporary* only, and necessarily ceased with the cause that produced them. They were sufficient however to *prove* to the society the absurdity of their plan, had they been disposed to profit by experience. Of late years, the dividend has been gradually decreasing; but it is *now* fixed that it shall not be less than £180.

admission, and the annual contributions for each share, to the age and other circumstances of the parties on whose lives the Assurance is made; with further power to grant (when applied for by the parties) Annuities in lieu of the amount insured, or in lieu of policies which it may be desirable no longer to continue. The *whole* of the annual contributions for every share granted previous to this charter, and *seven-eighths* of the contributions on shares subsequently granted, are to be divided every year amongst the claimants by deaths which happen in that year: and the corporation now engages that each share (although it may be more) shall not produce *less* than £180.*

The several charters provide that the joint stock,

* The following are some of the *Rates*, on which persons are now admitted to become shareholders, according to the ages of the lives assured.

Ages.	*Premium on Admission.* £ s. d.	*Annual Payments.* £ s. d.
15	1 : 11 : 6	3 : 10 : 6
20	1 : 14 : 6	4 : 0 : 6
25	1 : 17 : 0	4 : 9 : 0
30	1 : 19 : 6	4 : 19 : 0
35	2 : 2 : 6	5 : 11 : 6
40	2 : 5 : 0	6 : 7 : 6
45	2 : 9 : 0	7 : 7 : 0
50	2 : 13 : 0	8 : 12 : 0
55	2 : 16 : 6	10 : 4 : 0
60	3 : 1 : 6	12 : 6 : 0
65	3 : 6 : 6	15 : 6 : 0

The annual payments are to be made *half* yearly; and the first half yearly payment is to be made at the time of admission.

raised by the accumulation of the reserve found necessary for the security of this guarantee, is to be improved for the benefit of all the members, according to their respective interests: and no division of this capital is to be made, otherwise than in the payment of claims on death, without the consent of two successive General Courts of the members of the corporation. And the Society is now enabled to effect assurances *for any number of years*, or on the *joint* continuance of lives, or on *survivorships*, or on *any other contingency of life.**

The Royal Exchange, and London Assurance Companies.

§ 439. The *Royal Exchange Assurance* Company was formed about the time of the South Sea Bubble; and is one of the few projects which has survived that troublesome period,† and preserved its credit to the present day. It was established by charter dated June 22, 1720, under the name of *The Royal Exchange*

* No table has yet been published of the terms on which *these* Assurances are effected.

† Out of above *two hundred* visionary schemes, that were formed and *carried into effect* at that time, only *four* exist at present! viz. the Royal Exchange Assurance Company, the London Assurance Company, the York Buildings Company, and the English Copper Company. See Macpherson's *Annals of Commerce*, vol. iii. page 99.

Assurance, for insuring ships and goods at sea, or going to sea, and for lending money on bottomry. This charter was granted by virtue of an Act of Parliament (6 Geo. I, c. 18): but in the following year (April 29, 1721) the king (by virtue of his prerogative) granted them another charter for the Assurance of *Lives* and against casualties and accidents by fire, under the name of *The Royal Exchange Assurance of Houses and Goods from Fire.*

The *London Assurance* Company had a similar origin, and was incorporated in consequence of the Act above mentioned. It is also empowered to grant Assurances on Lives.

In addition however to the powers and privileges, which are common to both these Companies, the *Royal Exchange* was further authorised and enabled (by Act 33 Geo. III, c. 14) to sell and purchase *Annuities* either immediate or reversionary, under certain restrictions: and for the purposes of that act only they are (over and above the names by which they were before styled) to assume the name of *The Royal Exchange Assurance Annuity Company.* In order to render full security to the *Annuitants* under this act, it is provided that the money received for the annuity shall (as often as it shall amount to £1000) be laid out in government security; the interest or dividends on which, the Corporation may from time to time receive: but they are prohibited from touching any part of the *principal* till the extinction of the life on which the annuity is granted. And for the sake of preventing

any fraud in this respect, it is enacted that the time and place of the death and burial of each annuitant shall be inserted twice at least in the London Gazette: and that a certificate of such death or burial, with an account of the annuity depending, shall be left at the Office in London; to be inspected, without fee or reward, by any person interested in any annuity granted by virtue of that act. And it is likewise further enacted, that no agreement for the *selling* or *purchasing* of any annuity shall be valid, unless at the price stated in a Table which is directed to be hung up in some conspicuous place in the Office; and which table shall be remaining in the office at the time of granting the annuity.

It is difficult to discover upon what principles this Corporation has deduced its table of the *Rates* of Life Annuities.* Those rates have certainly not been deduced from any observations hitherto *published*; neither do they agree with any *probable* rate of human mortality: but seem to have been formed at random, without any regard to the true principles of the science. The values for all the lives *under* 50 years of age are much too *low*, even when compared with *M. De Parcieux's* Observations, at 4 per cent: but, *above* that age, they err *considerably more* from the true value, in the *contrary* extreme. If however their rates be

* Their rates of Life *Assurances*, as well as those of the *London Assurance* Company, are the same as those adopted by *all* the other Offices: of which I shall speak hereafter.

compared with those proposed by Government (as explained at the end of this chapter), they will be found *altogether too high:* and few persons, who are enabled to accept the proposals there alluded to, will be disposed to sacrifice so considerable a difference by *purchasing* an annuity at the Office of the Royal Exchange Company: or indeed at any other Office whose terms have yet been published.

Equitable Society.

§ 440. These were the only three Companies for granting Life Assurances till the establishment of the *Equitable Society* in 1762;* which was formed in consequence of Mr. Simpson's lectures, recommending such an institution. It appears likewise that Mr. Dodson was active in recommending the plan of this society; and composed some tables for its use. It is truly deserving the name which it has assumed: it being certainly one of the most *Equitable,* as well as the most important, of all the societies that have ever yet been formed for the purpose of granting Assurances on lives.

* Several Societies however had been formed, and still exist, for the purpose of assuring the lives of a *particular class* of the community; such as Naval and Military men, Clergymen, Schoolmasters and others. But it is not my intention to enter into any inquiry on the principles whereon *these* Societies have been established.

By its constitution, the assured are *mutual* assurers one to the other; and participate equally* in all the profits and advantages of the concern. These profits are estimated from certain periodical† valuations of all the claims upon the society, compared with its present assets.‡ *One third* of the clear *surplus* stock is then set apart as a fund for the permanency and future security of the concern.§ But, the remaining *two thirds* are divided amongst the assured; and the amount of each person's share (estimated in proportion to the time that he has been a member) is added to his policy: so that his representatives will, at his de-

* This applies only to those persons who are assured for the *whole continuance* of their lives: that *right* has not yet been extended to those who are assured for a *term of years*, except a Bonus should happen to be declared during the term for which the assurance is made; and then only in case the life should happen to become extinct *between the time of declaring such bonus and the end of the given term:* which, in most ordinary cases, can be but a *partial* advantage.

† The period of *ten* years (which has been adopted by the Society) is certainly too great an interval; since few persons can expect to enjoy the *full* value of their interest in the concern. A new plan, however, has just been adopted by the Society (which I shall presently mention) that will in some measure remove this objection, as far as the *old* members are concerned.

‡ See the method of making these valuations in Question XXXIV, page 457.

§ When it is considered that this *reserved third* is continually improved at interest, to the end of the next period, for the use of all the members, and then merges into the general stock to be again divided; it will appear that a person assuring at the Equitable Society, in its present established state (with its *reserved third* in abeyance), receives nearly the *whole* of his share of the *current* profits of the concern.

cease, probably receive *considerably more* than the original sum insured. That this has hitherto been the case will appear from the following statement.

In the year 1786, the addition of $1\frac{1}{2}$ per cent *on the sum assured*, which had been made to the claims in 1782, for *every annual payment* prior to that year, was increased to $2\frac{1}{2}$ per cent. In the years 1791, 1793, and 1795, still further additions were made; amounting together to 4 per cent. In the year 1800, another addition, of 2 per cent, was made to the claims: and we have just seen a still further bonus declared, of $2\frac{1}{2}$ per cent per annum, on all claims from assurances effected before December 7, 1809. So that at this time, the assurances of the year 1800 are increased by the *addition* of 25 per cent; those of 1790 are increased by 86 per cent; those of 1780 are increased by 180 per cent; those of 1770 are increased by 290 per cent; and those of 1762 are increased by 378 per cent. Or, in other words, (for I cannot be too explicit upon this important subject,) the executors of a person assured at this Office in the year 1800, for the sum of £1000 only, will on his decease be entitled to the sum of £1250: but, if the assurance has been effected in 1790, they will be entitled to £1860: if in 1780, to £2800: if in 1770, to £3900: and if in 1762 (when the Society was first established) they will be entitled to the enormous sum of £4780. Now, at *no other* Office (with the exception of the partial advantages derived from assuring at one or two of the Societies hereafter mentioned) would the executors be entitled to receive a shilling more than the sum of £1000 originally assured. Surely these *facts*

speak sufficiently for themselves: and show, most decidedly, the impolicy of assuring at those Offices whose sole object is gain, and who consequently make no return of their immense premiums.

But the most important alteration in the plan of this Society was that which was brought forward and proposed at the General Court held December 7, 1808: where it was resolved that in future the addition of 2 per cent on the sum assured should be made to each existing policy at the end of every year, for the next 10 years: at the end of which period another valuation is to take place, when the surplus profits of the Society (after allowing for the bonus just alluded to) will be divided amongst the existing policies, in the usual manner. So numerous however were the applications for admission into the Society, on this proposal being made known to the public, that it was thought necessary and proper to amend it. At the next General Court (December 19, 1809) it was therefore resolved to limit this advantage to the persons *already* assured at the Office, and to such claims *only* as should arise prior to the next division of profits. Consequently, by this new plan, the executors of a person already assured at this Office will be entitled to receive (in addition to any other bonus which may have been declared) 2 per cent (on the original sum assured) for every year which has transpired since the date of the last division of profits and the time of his death. But, this benefit is not *yet* extended to members effecting assurances *subsequent* to the date above mentioned.

The proceedings of this Society have hitherto been

founded on a firm and durable basis: and the Directors* have, in their management of it, shown the greatest prudence and discretion. Guiding themselves by the light of mathematical science (without which no establishment of this kind can be ever upheld) they have taken no step which can at all hazard its permanency or safety: 'and, in order to secure the same prudence and discretion in the future conduct of its members, the Society has provided by certain laws, "That no allowance to claimants shall ever be made "without a previous investigation of its affairs, and "the concurrence of *four fifths* of its members at "three successive General Courts; and also that such "allowance shall in no instance exceed *two thirds* of "the clear surplus stock of the Society." By these restrictions, as far as human precaution can operate, the danger of intemperate measures is avoided: and there is every reason to hope that an institution, founded on such liberal principles and defended by such wholesome provisions, will not only be secure, but continue to improve in credit and prosperity.†'

* The Directors are chosen annually by the members of the Society: and four General Courts are held in the year; at which are exhibited accounts of the state of the Company, and at which are formed such rules and regulations as may be deemed necessary for its advancement. But, by a law made in January 1810, no person, admitted a member subsequent to December 19, 1809, has a voice at the general meetings unless he has been assured for 5 years in the sum of £2000 on the whole continuance of a life. Nor can any such person be chosen a director unless he has been assured for 5 years in the sum of £5000.

† See the history of this Society, brought down to the present time, in Price's *Obs. on Rev. Pay.* vol. 1, Chapter II, Section 7.

Other Societies.

§ 441. Soon after the establishment of the Equitable Assurance Company a number of societies sprang up, (all about the same time*), which assumed the false and alluring title of Institutions "for the *benefit* of Old Age." These institutions were, for the most part, gross impositions on the public; proceeding from ignorance or knavery, and encouraged by credulity and folly. The promoters of them were principally persons in the more advanced ages of life, and who imagined the schemes would last *their* time: they were consequently little anxious about the *durability* of the plan, and desirous only of getting a *numerous* body of subscribers, that their plunder might be the greater. Agreeably to this principle, the founders of these societies began so low as not to require so much as a *fourth* or a *fifth* part of the values of the annuities which they undertook to pay: and encouragement was also held out to *early* applicants. These base attempts to deceive and allure the ignorant and distressed (and which unfortunately succeeded too well) first induced

* The Laudable Society, the Amicable Society of Annuitants, the Provident Society, the London Annuity Society, the Equitable Society of Annuitants, the Westminster Union Society, the London Union Society, the Consolidated Union Society, the Public Annuitant Society, the Rational Society, the Friendly Society of Annuitants, and many others (whose names are now lost or forgotten) were all established about the year 1770 and 1771.

Dr. Price to publish his *Observations on Reversionary Payments.* "On the first appearance however of this "work, the rage for establishing *new* Societies imme- "diately subsided; a *partial* reformation took place "in some of those which had been already formed: "and in a short time the greater part of them, con- "vinced of their mistakes, dissolved themselves. A "few, indeed, persevered in an obstinate adherence "to their original plans; but they have long since "exhibited a melancholy proof of their own folly, "and of the truth and justice of the admonitions which "had been wasted upon them.*"

The general practice of the promoters of those schemes was to attract the weak and credulous by holding out the most easy and advantageous terms. These arts succeeded wonderfully well at first; till the public became better informed on the subject; when they soon fell into disrepute and contempt. But although such specious proposals should always be viewed with suspicion, the public should be cautious not to fall into the opposite extreme, and suffer themselves to be enticed into the payment of exorbitant rates, under the delusive plea of stability and security. They ought to satisfy themselves that any extra charge which is made upon them (over and above the fair value, after allowing a proper and liberal compensation to the proprietors) is *sacredly* and *inviolably* appropriated to secure the permanency of the establishment, and to

* *Introd.* to Dr. Price's *Obs. on Rev. Pay.*

provide for those unfavourable events which sometimes arise and overturn the best contrived schemes.

Towards the end of the last, and the beginning of the present century, several new Public Companies were formed for the purpose of making Assurances on Lives, granting Annuities, &c: viz the *Westminster Society* in 1792; the *Pelican* in 1797; the *Globe* in 1799; the *Albion* in 1805; the *Rock* and the *Provident Institution* in 1806; and the *Eagle*, the *Hope*, the *London Life Association* and the *Atlas* in 1807. None of these societies however are confined *solely* to the granting of *Life* Assurances: since they all unite, thereto, the purchase or sale of *Annuities*; and five of them (in addition to the other two branches) are engaged in *Fire* Assurances. It is not my intention to enter into the nature and form of *all* these different Societies; since the major part of them do not (as far as the subject of this work is concerned) profess to hold out any peculiar advantages to the *public*.* But, as *two* of them in particular differ materially in this point from all the rest, I shall devote a few lines to their separate explanation and examination.

The Provident Institution.

§ 442. The *Provident Institution* was formed in 1806, with a view "to enable the industrious and economical

* There is some trifling difference between them as to the *fees on admission*, and as to *military* persons: but, these are *paltry* considerations compared with those which I shall hereafter mention.

"to appropriate their savings most beneficially to their "different objects of prudence or affection; and to "encourage that laudable disposition by affording an "ample guarantee for the fulfilment of the benefits "proposed, *without curtailing them by commercial* "*views of profit.* With these objects *several hundred* "noblemen and gentlemen have entered into a sub-"scription of £250,000 to constitute an original ca-"pital *; and taking upon themselves the sole re-"sponsibility of the establishment, they render the in-"sured and annuitants *proprietors,* and entitle them "*equally* to participate in profits, after the expenses "of an economical arrangement are defrayed, in which "the Presidents, Directors, Trustees and Auditors "act gratuitously. Thus, by the specific sum insured, "and the division of contingent profits, every member "of this institution has the *fullest value possible* †, for "his payments, without being subject to any calls or "risk whatever."

This institution not only makes Assurances on lives, but also buys and sells Annuities, and grants Endowments for children. Their proposals, at first sight, appear to be the same as in the Equitable Society; since all the proprietors are stated to participate *equally* in the profits of the concern. There is however this material difference, that, in the Provident Institution,

* These subscribers are not *obliged* to insure *their own* lives, as is the case at *all the other* newly established Offices.

† This is a *false* assertion, as will evidently appear in the sequel.

those profits are shared in conjunction with the *several hundred* noblemen and gentlemen who have *guaranteed* * the capital; and who appear not to have wholly lost sight of the *commercial views of profit* in establishing this concern †.

The Rock.

§ 443. The *Rock Life Assurance Company* was formed also in the year 1806; and appears in some respects to have followed the plan of the Equitable Society: it is however in its nature essentially different. In the Rock, the assured are not *mutual* assurers one to the other: neither do they participate *equally* in the profits of the concern. For, though at certain periods (of not less than seven years ‡) an estimate is made of those profits, and two thirds of the same, *after deducting the sum of five thousand pounds therefrom* §, are divided amongst all the policies, as in the Equitable Society; yet the remaining third, instead of being improved for the benefit of *all* the parties, is appropriated

* A small portion only of the capital has been actually *advanced.*

† See some observations on the *Rates* of this Society in § 374 and 446.

‡ The *first* division of profits, however, is not to take place till after the expiration of *twelve* years from the establishment of the Society.

§ This is provided for, in the Deed of Settlement, although it is not mentioned in the Prospectus of the Company. See a pamphlet issued by this Society under the erroneous and illusive title of *The Principles of Life Assurance explained.*

to the use of the *proprietors only*. This third part is thus for ever *lost* to the rest of the assured; and, at the end of the next period, another third part is likewise taken away: which *continual subtraction* operates in the inverse method to that pursued by the Equitable Society, and must in a course of years make a very *material* difference in the amount to be added to the policies at each respective office.*

General Observations.

§ 444. With respect to the *remaining* institutions of this kind now in existence, I believe they are all formed with a view to *private* interest, and do not profess to hold out any peculiar advantage to the *public*.† It will be recollected that, a few years ago, there existed in this country a great rage for *Joint-Stock Companies*; and the public were much amused with the various proposals that daily issued from the press. The enormous profits that are made, by granting Life Assurances and Annuities, attracted the hopes and expectations of many persons; and several of the more recent Offices may ascribe their origin to these popular impressions. The success, attending the establishment of these

* It will appear from this view of the subject that a person in suring at the Rock does not stand a chance of receiving even on *half* the additional profit that he would probably obtain by insurir at the Equitable. See the note in page 487.

† See the note in page 493.

Offices, and of similar undertakings, encouraged others to come forward, not only with the like proposals, but with various fanciful schemes and visionary projects: and there was every appearance that the times of the South-Sea Bubble were about to be revived. The same measures, however, that put a stop to the nefarious practices of that period, were now resorted to: the act of 6 Geo. I, c. 18,* after lying dormant above 80 years, was applied to the correction of this public grievance; and the decision of the Court of King's Bench, in the case of *The King* versus *Dodd*, put a stop to all these delusive projects. As the opinion of the Lord Chief Justice (Ellenborough) on that occasion cannot be made too public, and as it has some connection with the present subject, I shall make no apology for here inserting it.

"This case has been very fully argued, and the application for an information has at least had this

* By that statute it was declared that all the mischievous undertakings and attempts *therein described*, tending to the common grievance, prejudice and inconvenience of his Majesty's subjects, in their trade and commerce,—and all public subscriptions, receipts, payments, assignments or transfers, for the purpose of furthering, countenancing, or proceeding in such undertakings,—and more particularly the presuming to act as a corporate body, and pretending to raise and transfer any shares in such concerns without legal authority (either by Act of Parliament or Royal Charter)—should be illegal and void. All such undertakings are declared to be a public nuisance; and persons offending therein are not only subject to the usual fines in such cases, but are further liable to the penalties and forfeiture of the statute of *Præmunire*.

" good effect, that it has produced a full discussion
" of the question, and has given a general notoriety
" to the existence of the statute of the 6 Geo. I: so
" that no person can hereafter pretend to say that it
" is an obsolete law; and, on that account, no longer
" to be enforced against such as offend against the
" provisions of it. After a lapse, however, of eighty-
" seven years since any authenticated proceeding has
" been had upon this branch of the act, and when
" other ways are still open to the party now applying
" to put this act in force against offenders, the Court
" (in the exercise of a sound discretion, under all the
" circumstances of the case) will forbear to interfere
" in this extraordinary manner. But, at the same
" time we wish it to be understood that it is not be-
" cause we think that the facts brought before us are
" not within the penalty of the law: but we choose to
" express ourselves with the greater reserve, because
" the defendant may still be indicted, and the Court
" may still be called (upon the removal of the indict-
" ment by *certiorari*, or upon an information filed by
" the Attorney-General) to give their opinion on this
" very case. But (independent of the general tendency
" of schemes, of the nature of the project now before
" us, to occasion prejudice to the public) there is be-
" sides in this prospectus a prominent feature of mis-
" chief: for, it therein appears to be held out that no
" person is to be accountable beyond the amount of
" the share for which he shall subscribe, the conditions
" of which are to be included in a Deed of Trust to

"be enrolled. But this is a *mischievous delusion*, "calculated to *ensnare* the unwary public. As to "the subscribers themselves, indeed, they may stipu- "late with *each other* for this contracted responsibility; "but as to the rest of the world, it is clear that *each* "partner is liable to the *whole amount of the debts* "*contracted by the partnership*. I forbear to com- "ment on lesser circumstances; such as the smallness "of the sum to be subscribed in the first instance,* "(which seems to carry an appearance of holding out "a *lure to the unwary*) and other features in the "case. But (considering that this is brought forward "after a lapse of so many years since any similar pro- "secution was instituted, and brought forward by a "party who does not profess to have been himself de- "luded by the project; and the statute having been "passed principally for the protection of unwary per- "sons from delusions of this kind) the Court think, "in the exercise of their discretion, that they should

* Of all the Life Assurance Companies that have lately been established, the *Globe* is the only one that has paid up its subscription in *full*. The remaining ones have divided their capital into small shares, on which 10 per cent only has been paid. Thus, the *Eagle* professes to have a capital of Two Millions; whereas it in reality consists of no more than £200,000, which is divided into 40,000 shares of £5 each. And the *Rock*, which professes to have a capital of One Million, is divided into 50,000 shares of £2 each. A similar observation will apply to most of the other Societies. It is true that they have reserved a privilege of making a *call* for the remainder: but I hope no circumstances will ever arise, wherein their power or pretensions may be *disputed*.

"not now enforce the statute against this defendant, "at the relation of a person so circumstanced: leaving "the relator to the common law remedy by indict- "ment; or the defendant to be proceeded against "by His Majesty's Attorney-General *ex officio*, if he "should deem it adviseable for the protection of the "public. But, the Court think it is fit that this rule "should be discharged without costs. And they re- "commend it as a matter of prudence to the parties "concerned, that they should forbear to carry into "execution this mischievous project, or any other "speculative project of the like nature, *founded on* "*joint-stock and transferable shares:* and we hope "that this intimation will prevent others from engaging "in the like *mischievous and illegal projects.**"

With respect to the constitutions of the several Assurance Companies above alluded to, they are all of them (with the exception of the *Amicable* and *Equitable* Societies) in one particular nearly the same. In each of them the Assured may be divided into two classes, Proprietors and Non-Proprietors. The former are those who hold shares in the concern and who divide all† the profit among themselves: and the latter

* East's Reports, in Easter Term 1808, vol. ix. page 525.

† This must be taken with some limitation as far as it regards the *Provident Institution* and the *Rock*; agreeably to what I have above stated, when speaking of those companies. There is also the following advantage held out by the *Hope* Insurance Company: viz. "Persons, not being proprietors, who insure for £500 upon "their own lives, will, upon the payment of ten shillings per cent

consist of such other persons as have been induced from *ignorance, persuasion, or necessity* to effect assurances at those Offices; and who (although they pay the same premium on their policy as at the Equitable Society) will never receive a shilling more than the sum originally insured.

For, it is necessary here to remark (and I think it an observation of considerable importance) that the *rates* of Assurances on lives do not differ, in the least, at any of the Offices: so that whether a person makes an assurance at the Equitable Society where the sum assured is continually *increasing* in value,* or whether he effects it at any other Office where no additional advantage is derived, *he pays precisely the same premium!* Surely this important fact cannot be sufficiently known by the public, else it is difficult to conceive how any of the newly established Offices should ever have been able to extend their business beyond the limits of their own proprietary.†

" (on their admission) beyond the premium paid for the Assurance, " be entitled, out of the profits of the Life Fund, to such interest " or dividend as may be paid on a £50 share to the several pro- " prietors thereof; and so on in like manner for every £500 so " insured. But in no case whatever are they to be responsible " for any claims which the Life Fund may be called upon to pay; " the Capital of the Company being of sufficient magnitude for " that purpose."

* See page 488.

† Every *proprietor*, in all these newly established Societies, is *obliged* to insure a given sum on his own, or on some other person's life.

Independent, however, of the superior *advantage* of insuring at the Equitable Society, there is a satisfaction arising from the consciousness of its superior *security*.* This security originated from, and is principally maintained by, the power which each member possesses (in his own right) of giving his vote and opinion on all the proceedings of the Society: so that no unfair advantage can be taken by any particular class of the assured. On the contrary, at all the other Offices† the management of the concern is in the hands of the Proprietors *only:* and the Non-Proprietors (or those who assure at the office without holding any share) have no voice whatever in the administration of the affairs of the Company. They may see the proprietors voting large dividends amongst themselves, and the capital of the society (which ought to be improved with the greatest economy) annually dwindling away: but, though this should be apparent to the most common observer, they would be, in a great

* A person who makes an Assurance against *Fire* is content if the Office, at which he insures, possesses the *present* confidence of the public and of himself: since he may remove his policy to any other Office *whenever* he doubts its stability or responsibleness. Not so with a *Life* assurance: for, a policy of that kind, once entered into, cannot be removed without considerable loss and inconvenience to the party; and, as the *claim* may not become due for 20, 30 or 40 years (nay, perhaps for more than half a century) it therefore particularly behoves him to be satisfied not only with the *present* flourishing state of the society but with the prospect of its *permanency* and *future solvency*.

† Excepting the Amicable Society.

measure, unable to relieve themselves. For, most persons, after continuing the payments on their policies through a series of years, would think it too serious an injury to withdraw; and would rather risk the issue of the event.

Now, though I by no means wish or intend to cast any reflection on the present managers of any of the more recent Societies, many of whom I *know* to be men of strict honour and integrity, and who (considering themselves equally trustees for the *public* as for the *Society*) would not give their sanction to any measure that was likely to endanger its security, or hazard its reputation; yet *their* voice may not always prevail against a numerous body of proprietors, neither can *their* services be perpetuated. And the sad experience of almost all preceding establishments of a similar kind, which have been formed with a view to the benefit of posterity, shows that this is not an *imaginary* evil only:* and that it becomes *every* person connected with such Societies to be particularly cautious that no attempts are made, either through ignorance or design, to risk the security of the establishment by grasping too eagerly at immediate or enormous profits. They should bear in mind the advice that was given to the Directors of the Equitable Society by a celebrated

* Let any person read the nefarious practices alluded to by Dr. Price in his *Observations on Rev. Pay.* and by Mr. Dale in his *Calculations deduced from first Principles;* he will then be convinced that the members cannot keep too vigilant an eye over the proceedings of their society.

writer on these subjects, and the adoption of which has tended to raise that establishment to its present state of eminence. "They should consider what "distress would arise from the failure of the concern "at any future time; and what are the dangers which "ought to be carefully guarded against in order to "secure success."——"That it is not to be expected "that any society can meet with difficulties in its in-"fancy; because, not till the end of many years, after "it has acquired its *maximum* of members, will the "*maximum* of yearly claimants and annuitants come "upon it. Should it, therefore, through inattention "to this remark, and the encouragement arising from "the possession of a large surplus, be led to check or "stop the increase of its stock [or to divide the pro-"fits of the concern] *too soon*, the consequences "might be highly pernicious.*"——"That it is of "great importance to the safety of such a society that "its affairs should be under the inspection of able

* The distinguishing principle of the *London Life Association* is, that the profits resulting from its transactions shall be shared by the members *during life:* whereby the security of that establishment must be considerably impaired. For, "in matters of chance "it is impossible to say that an unfavourable run of events will "not come, which may hurt the best contrived scheme. The cal-"culations only determine *probabilities*; and agreeably to these "it may be depended on that events will happen *on the whole*. But, "at *particular* periods, and in *particular* instances, great deviations "will often happen; and these deviations, at the commencement "of a scheme, must prove either very favourable or very unfa-"vourable."

"mathematicians. Melancholy experience shows that "none but mathematicians are qualified for forming "and conducting schemes of this kind. In short; "dangerous mistakes may sometimes be committed, "if the affairs of such a society are not managed fru-"gally, carefully, and prudently.*"

Rates for Life Assurances.

§ 445. I cannot conclude this chapter without some observations on the *Rates* of Life Assurances published by the several Companies above alluded to: and, as those rates are the *same* in all, my remarks will *equally* apply to each of those different societies; excepting perhaps the *Equitable*, the nature of whose establish-

* Price's *Obs. on Rev. Pay.* vol. 1, page 179. The same author (in pointing out, in another place, the dangers to which a Society of this kind is exposed) remarks, "that there are no questions, the "solution of which requires a stricter attention, or greater skill in "investigation, than some in the doctrine of Assurances. Difficult "questions are sometimes brought to the Society; and the Di-"rectors, not being themselves mathematicians, are under a ne-"cessity, in making their demands, of being governed by their "Actuary: and should he happen to be unqualified, he must make "mistakes, and either the Public or the Society will be injured. "In short, the Society can scarcely be sensible enough of the "importance of both abilities and probity in the servants it em-"ploys: nor, therefore, of the particular reason there is for guarding "their places against the applications of candidates who, on any "future vacancies, may endeavour to intrude themselves by their "connexions or influence."

ment renders it not of so much moment what the rates are, provided every member pays in proportion.* The promoters of that concern showed their sound judgement and good sense in adopting such tables of the probabilities of life as were rather *below* than *above* the truth. They first made use of Mr. Simpson's Table for *London*; but, finding these too inaccurate, they changed them for the *Northampton* Table. Experience has confirmed the propriety of their choice; but, at the same time, has proved to us that the *Northampton* Table is by no means a *corret* index of the rate of human mortality amongst the members of such a society.† It has, however, been adopted by all the *other* offices, under the delusive plea that "experience "has shown it to be the most just and liberal scale, "whereby to estimate a fair value between the two "parties." But, however applicable the *Northampton* Table may be to the use of a society *constituted as the*

* I should likewise except the *Amicable Society*, whose plan and constitution differ so materially from the others.

† Mr. Morgan says (Price's *Obs. on Rev. Pay.* vol. 1, page 183) that the decrements of life in the Equitable Society, from the year 1768 to 1800, compared with the decrements of life in the *Northampton* Table, appear to have been from the age of

10 to 20 in the ratio of	1 to 2
20 .. 30	1 to 2
30 .. 40	3 to 5
40 .. 50	3 to 5
50 .. 60	5 to 7
60 .. 80	4 to 5
All ages together	2 to 3

Equitable is, it certainly is not entitled to the same degree of approbation as affording a fair estimate of the value of annuities and assurances between the public and a *trading* company; nor even as between two individuals, unless under particular circumstances. A more just and equitable scale *ought* to be adopted by those societies who do not make any return to the assured, of the vast profits that arise from this species of daily traffic; and would tend more to the increase of their business in this way, and would likewise be more honourable to themselves, than the disgraceful practice of *bribing* solicitors, agents and others to effect assurances at their Offices: thereby notoriously inducing those parties to *sacrifice* the interest of their employers and their friends.* For, the money which is applied to this base purpose can be considered in

* Many of the public Companies, who do not make any return of the profits to the assured, allow a *liberal premium* (generally 5 per cent on the payment made) to any person who will procure an insurance to be effected at their Office: and this commission is *also* allowed to *any* person who makes the *annual* payment *provided it be not the party himself!!!* An artifice which is easily seen through: but which opens such a door to fraud and imposition, that it cannot be too severely reprobated. And however much it may be sanctioned by the Directors in their *public* capacity, we are all aware what their emotions would be if they discovered any of their tradesmen tampering with *their own* servants in this opprobrious manner: since they must well know *who* would eventually pay for it. I omit to give the names of those companies who have adopted this nefarious practice, under the hope that such a mean and improper artifice will not be encouraged in future.

no other light than as *unjustly* taken from the pockets of the *assured*; and would be more properly and more equitably employed in being appropriated towards a reduction of the rate of assurance: since, if the Company can afford to allow it to the *agent*, it surely can afford to allow it to the principal; and it evidently belongs more *justly* to the latter than to the former.

The Tables of the *Rates* of Life Assurances, as published by the several Companies, are precisely the *same* in each; and are usually *three* in number: viz. one for *Single* lives, one for *Joint* lives, and the other on the *Contingency* that one life shall die before another life.* These rates are all deduced from the *Northampton* observations, and at the rate of *three* per cent interest. By thus computing the values from the *lowest* probabilities of life, and at the *lowest* rate of interest, the Rates become, in most cases, full a *third* more than they ought to be when calculated from the more correct tables of the probabilities of life as observed in *Sweden*, at the rate of *four* per cent interest; and still more do they differ from the true value as deduced from the probabilities of life as observed by *M. De Parcieux*, at the same rate of interest.

* These Tables are LI, LII, and LIII at the end; and which may be had at any of the Assurance Offices. On a comparison of them, as issued by the different Offices, they will be found to be all alike: there being only a trifling difference of a *penny* or *twopence* per cent in any of them; which is sometimes in *excess* and sometimes in *defect*.

Now, since the rate of human mortality, amongst the members assured at such Societies, is by no means correctly indicated by the *Northampton* * table of observations, but approximates more nearly to the observations made in *Sweden*, or to those made by *M. De Parcieux*, the public may form a tolerably correct idea of the immense † profit that is made by this species of daily traffic; and may learn, to their own cost, the impolicy of insuring at those Offices whose sole object is gain, and who consequently make no return of any part of these exorbitant demands. A single instance will illustrate this.

A person aged 20 is desirous of assuring his own life for £5000: the sum demanded for this purpose by *all* the Assurance Companies is £2140: 2: 0. But the true value of such sum, as deduced from the *Sweden* Observations, is no more than £1422 : 10 : 0, if we take the rate of interest at 4 per cent; and no more than £1128: 10: 0, if we take the rate of interest at 5 per cent: or when deduced from the observations of *M. De Parcieux*, it is no more than £1358 : 2 : 0, taking interest at 4 per cent; and no more than £1078: 16: 0, taking interest at 5 per cent. A person, therefore, of this age who insures the above sum at any of those Offices, which make no return of any part of the pre-

* It is in the ratio of only 2 to 3. See the note in page 506.

† Some idea may be formed of these enormous gains, from the large sums which have occasionally been added to the policies at the Equitable Society; and which have amounted to above *one million and a half* in present money. See page 488.

mium, may be considered as *throwing away* between seven hundred and a thousand pounds *. I say that this sum is thrown away by insuring at *such* Offices, because, by effecting the same assurance at the *Equitable Society*, the money thus paid down is carried to a common stock, and the *whole* † of the profits of the concern (after the expenses of an economical establishment are defrayed) are from time to time divided amongst *all* the persons assured, and added to their policies: so that the executors of a person assured at *that* office have a chance of receiving (at his death) *considerably more* ‡ than the sum originally insured. The public, I presume, are not aware of this important fact; otherwise it is difficult to conceive how the other

* These calculations are deduced from the supposition that the *whole* premium is paid down immediately in *one payment*: but, nearly the same differences will arise if we suppose the premiums to be made *annually*. For the annual payment which would be required, by all the Offices, for the assurance above alluded to, is £108 : 19 : 0: whereas the value of the same, deduced from the observations of *M. De Parcieux*, is £71 : 14 : 3, if interest be taken at 4 per cent, and only £65 : 10 : 0, if interest be taken at 5 per cent.

† Every person now insuring at that Office may be considered as partaking of the *whole* of the profits of the concern; for the reasons given in the note in page 487.

‡ They have a chance of receiving considerably more, for two reasons: in the first place the Society can generally make more than *three* per cent interest of their money; and secondly, the lives do not die off so fast as the computations suppose. So that the money is not only improved at a better rate of interest, but also for a longer period.

Offices should ever have been able to effect a single policy, except such as the Equitable Society had refused.

The same observations will apply to the case of *Joint* lives; since the values are here also deduced from the *Northampton* table of observations, and at the rate of *three* per cent interest. But, if we take the case of *Contingent* assurances (that is, of an assurance made on a given life, or the contingency that it dies before another) we shall find still greater cause for censure. For, independent of the rates being computed from the *lowest* probabilites of life and at the *lowest* rate of interest, they are also deduced from a rule given by Mr. Simpson *, which produces a result that is oftentimes more than *one third* of the true value too much, even when computed from the *Northampton* table, at 3 per cent interest!—A single instance will confirm this also.

A person, 10 years of age, is desirous of assuring the sum of £100 on his life, on the contingency that he dies before another person aged 60. The sum which would be demanded by *all* the Assurance Offices (not even excepting the *Equitable*) is £12 : 18 : 6 in a single payment, or £1 : 6 : 0 in annual payments during the joint lives. But the true value which *ought* to be given for the same (even on the supposition that the rate of interest is *no more* than 3 per cent, and the probabilities of living the *same* as observed at *North-*

* See the Scholium in page 446.

ampton) is only £10 : 13 : 0 * in a *single* payment, and £1 : 1 : 5 in *annual* payments: consequently the Offices demand about a *fourth* part of the true value *more* than (*on their own data*) is just and equitable. If the value, however, had been deduced from the probabilities of living as observed by *M. De Parcieux*, and at the rate of 4½ per cent interest, it would be £9 : 3 : 1 in a single payment, and only *eighteen shillings and ninepence* in annual payments: which makes the Office rate, in this case, nearly one *half* the true value too much!

Mr. Morgan has taken considerable pains to prove that Mr. Simpson's rule for finding the value of these contingent reversions is *exceedingly defective* †; and that it oftentimes leads to conclusions too erroneous to be overlooked. Now, since the *true* values can in all cases be obtained with so little trouble, it is somewhat singular that the incorrect values, in Table LIII above alluded to, should still be adopted (not only by the *Equitable Society*, but also by every other Office in London) for the purpose of determining the sums that

* I take this sum from Mr. Morgan's calculation in *Phil. Trans.* for 1788, page 344; who says that he has compared it with the sum deduced from the *exact* values of annuities on the joint lives involved in the solution.

† See *Phil. Trans.* ibid. The *singular* formula however, from which Mr. Morgan has deduced his values, has at length been amended by him in the *last* edition of Dr. Price's *Obs. on Rev. Pay.* Note (O). See what has been said in the note in page 187 of the present work.

are required for effecting assurances on the contingency here mentioned. Amongst the numerous societies that have lately been established, is there no one Actuary that has the confidence to propose a *new* table of the value of such assurances, *founded on a true and proper basis:* or *will* the several Companies still persevere in their unjust and illiberal demands? Surely their profits must be sufficiently great by taking the *lowest* rate of interest, and the *lowest* probabilities of living as the basis of their calculations; without adding thereto the *unfair* advantage arising from the use of *inaccurate rules* *.

The following tables (given by Mr. Morgan himself †) of the present value of £100 payable on the decease of A, provided B be then alive, will show "how far Mr. Simpson's approximation, the only rule "now in use [*by the Assurance Offices*], may be de-"pended upon."

* These observations will, in a great measure, apply also to the Equitable Society; since those, who assure on this contingency, pay more *in proportion* than the rest of the members: and it is curious to observe how servilely the *other* Offices have followed their example. See what has been already said upon this subject in the note in page 188.

† See *Phil. Trans.* ibid.

Interest 3 per cent, *Northampton* Observations.

Age of A	Age of B	Value by Simpson's rule.	Value by correct rule.	Age of A	Age of B	Value by Simpson's rule.	Value by correct rule.
10	10	24·75	24·75	20	20	27·96	27·96
10	20	23·50	22·11	20	50	19·29	18·65
10	30	21·47	19·84	30	30	30·21	30·21
10	40	19·07	17·10	30	60	18·19	17·51
10	50	16·21	14·04	40	40	32·87	32·87
10	60	12·93	10·65	40	60	22·78	21·92
10	70	9·15	7·07	40	70	15·78	15·35

Interest 4 per cent, *Sweden* Observations.

Age of A	Age of B	Value by Simpson's rule.	Value by correct rule.	Age of A	Age of B	Value by Simpson's rule.	Value by correct rule.
14	20	17·82	15·42	40	40	26·99	26·99
16	40	16·23	13·71	40	76	9·81	9·21
20	20	19·84	19·84	42	60	19·58	16·11
24	60	13·01	9·39	52	76	14·00	12·58
28	40	20·44	17·60	60	60	36·34	36·34
36	60	16·81	12·29	64	76	22·81	23·81

It will be seen by this comparison that, when the ages of the two lives are *equal*, the exact values are found by either rule *: but when there is any considerable inequality between the ages of the two lives, the values by Mr. Simpson's rule are in many cases

* Owing to some strange error in his calculation, Mr. Morgan has made the values deduced from Mr. Simpson's formula (according to the Sweden observations and when the lives are equal) *different* from those obtained by the correct rule: whereas it must be evident, from an inspection of the formula in the note in page 446, that they ought in such case to be *precisely the same*. The correction of this error is the only alteration I have made in the Table.

so wide from the truth, as to show the impropriety of their being used by any office, *professing* to be guided by mathematical principles.

Rates of Life Annuities.

§ 446. The observations which I have made in page 361 will apply with equal force to the terms on which the *Provident Institution* grants *deferred* Life annuities. For instance: the sum of £116 : 8 : 0 is demanded for an annuity of £20 on the life of a person, now aged 20, to commence when he is 40 years of age. Now the value of the same annuity, on the life of a person aged 40, is (at the same Office) estimated at £296. But £116 : 8 : 0 put out to interest at 5 per cent for 20 years would amount to £308 : 16 : 10; or to near £13 *more* than the sum for which he might *then* obtain the same annuity, without the risk of losing his money *in the mean time !!!*

With respect to the value of Life Annuities in general, at this Office, (and which is the only one, except the Royal Exchange, that has hitherto published its rates), I would observe that, like the values adopted by that Corporation, they are too *low* in the younger, and too *high* in the older periods of life. The cause of which anomaly is too apparent: since life annuities are generally purchased by those of more *advanced ages*, and are seldom resorted to by the *young*. I would however remark that those persons who are desirous of *purchasing* annuities on their own lives, or on any

other person's life, or on two lives with benefit of survivorship, have now * an opportunity of doing it with much greater security and advantage, than at *any* of the Offices above alluded to. For, by the act 48 Geo. III, c. 142, any person may exchange *Reduced* or *Consolidated* 3 per cent stock for an annuity on the life of a nominee not younger than 35 and not older than 75 years of age. And as the terms, on which this exchange can be made, are always more advantageous to the party than by a direct purchase of such annuities at any of the Offices; it is probable that this plan will, for the future, be the only one resorted to, in such cases, as long as it remains in effect †. This observa-

* In my *Doctrine of Interest and Annuities*, page 106, I suggested the propriety and advantage to Government in exchanging some of the Public Stocks for *Life* Annuities: and I proposed to enter more at large on this subject at some future opportunity. But, as that plan has since been adopted on so fair and liberal a scale, I can only add my best wishes for its success.

† The *rate* at which the exchange can be made depends on the price of the stock on the day *preceding* the day of transfer: and the exchange may be effected at any time when the price of the 3 per cents is between 60 and 81; the terms varying according to every *one per cent* difference in the market price of the stock. The limits of this work will not enable me to insert the whole of the Table given in the act: I shall therefore subjoin only a few of the values of annuities on Single lives according to *three* different prices of stock; to which I shall add the terms of the Royal Exchange Assurance Annuity Company and of the Provident Institution (the only ones that have hitherto published their rates), whereby an estimate may be formed of their comparative advantages. I would previously, however, observe that the annuities granted by Government and by the Provident Institution are pay-

tion, however, applies only to such persons as wish to *purchase* annuities. But there are many others who are obliged, from various causes, to *sell* or grant annuities on their own, or on other persons', lives: and several of the Offices are in the habit of purchasing the same; or, of advancing money on such contingencies. I am unacquainted with the *terms* on which this part of their business is conducted: neither do I suppose that they are nicely weighed by that class of persons who are driven to such an unfortunate and ruinous measure,

Conclusion.

§ 447. Before I conclude this subject I ought to mention that, in consequence of the pernicious practice of raising money by the sale of Life Annuities,

able *half-yearly:* whereas those granted by the Royal Exchange are payable *quarterly*. But this does not make any *material* difference in the true value.

Age.	Royal Exchange Rates.	Provident Institution Rates.	Government Rates.		
			4 per cent. or Stock at 75.	4½ per cent. or Stock 66⅔.	5 per cent. or Stock at 60.
35	15·380	15·600	14·563	13·878	13·043
40	14·700	14·800	13·761	13·072	12·371
45	13·700	13 700	12·821	12·232	11·650
50	12·820	12·600	11·719	11·299	10·714
55	12·050	11·495	10·638	10·256	9·836
60	10·870	10·250	9·494	9·009	8·889
65	9·800	9·000	8·242	8·081	7·792
70	8·770	7·750	6·849	6·734	6·522
75	7·750	6·500	5·395	5·333	5·195

which was much promoted by the secrecy with which such transactions were conducted, it was enacted by 17 Geo. III, c. 26 (commonly called the *Annuity Act*) that a memorial of all deeds, bonds, &c. for granting Life Annuities shall, within 20 days of the execution thereof, be enrolled in the Court of Chancery; which memorial shall not only contain the date, names of *all the parties* (and for whom any of them are trustees), and of all the witnesses, but shall likewise set forth the amount of the annuity, the consideration (which shall be in money only *), and the name of the annuitant: otherwise such deed, &c. shall be null and void †.

This being the case, it would be almost impossible for any of those societies, of the nature of Joint-Stock Companies, to deal in this kind of securities; since the names of *all the proprietors* (amounting to some hundreds, or perhaps to some thousands‡) must be inserted in the deed: and it would in fact be difficult, if not impossible, to collect all those names, at the time of making any one contract, owing to the fluctuating state of the proprietary. To obviate this inconvenience, some of those Societies have obtained Acts of Parlia-

* Bank notes, a cheque on a banker, a promissory note, or a bill of exchange are held to be a good consideration provided they are *paid when due*, and also provided that they are *fully set forth* in the deeds.

† There are some exceptions to this act; amongst which are annuities granted *by* any Corporate Body, or under any authority or trust created by Act of Parliament.

‡ See the note in page 499.

ment with a view to enable them to enrol such memorials in the names of the parties to such deeds, &c; and with an intention that such enrolment might (all other requisites of the Annuity Act being duly complied with) be as good and effectual, to all intents and purposes, as if the names of *all* the parties interested were inserted therein.

The Societies, that have obtained these illusive * Acts, are the *Globe*, the *Albion*, the *London Life Association*, the *Pelican*, and the *Provident Institution:* and there is a clause in those acts whereby these Companies are enabled to *prosecute* and to *sue* in the name of one of their officers; but they have not the privilege of *being sued* in the same manner. And it is

* See the Acts 47 Geo. III, Sess. 1, c. xxx, xxxi, xxxii, xxxiii and xxxiv. Owing to a strange oversight, these Acts became of no effect, almost as soon as formed; and, in order to render them of any real service to the parties, it was found necessary to have them amended. The *Globe*, the *Albion* and the *Pelican* therefore obtained new Acts for that purpose, in the very same year; being 47 Geo. III, Sess. 2, c. lxxxvi, lxxxvii and lxxxviii: but the *London Life Association* and the *Provident Institution* remain in their original doubtful state. These new Acts, however, being still open to some objections, were further amended and *explained* (at least, the titles *so* express it) by 49 Geo. III, c. cxxiii, cxxiv and cxxv.

Under shelter of these statutes, the *Albion* boldly claims the ambiguous quality of being "Empowered by Act of Parliament:" but the *Provident Institution*, with *less* right and more effrontery, falsely asserts that it is "specially empowered by Act of Parliament "for the Insurance of Lives, and the grant and purchase of Annui-"ties." Surely this artifice is unworthy the respectable names at the head of those Societies.

moreover expressly provided by each Act "that nothing "therein contained shall extend, or be deemed, con- "strued, or taken to extend to *incorporate* the said "society or partnership; or to relieve or discharge the "said society or partnership (or any of the members "thereof, or subscribers thereto respectively) from any "responsibility, contracts, duties or obligations what- "soever; which, by law, they may (now, or at any "time hereafter) be subject or liable to, either as be- "tween such society or partnership and others; or "between the (or any of the) individual members of "such society or partnership and others; or among "themselves; or in any other manner whatsoever."

I cannot close this chapter without observing that, by statute 14 Geo. III, c. 48, it is enacted that no Assurance shall be made on lives, wherein the party assured hath no interest; that in all policies the name of such interested party shall be inserted; and nothing more shall be recovered thereon than the amount of the interest of the assured. And it has lately been decided that, although an interest existed *up to the very time that the claim became due*, yet, if such interest be *afterwards* destroyed, the money could not be recovered. See (in East's Reports, Vol. 9, page 72) the case of *Godsall and others* versus *Boldero and others*, three of the directors of the *Pelican* Life Assurance Company, who refused to pay the sum of £500, insured on the life of the late Mr. Pitt, under the plea that his debts had been paid by Parliament.

TABLES.

TABLE I.

Showing the Number of Persons *living*, at the

Ages	Lond.	Stockholm		Vienna	Berlin	Nor-wich	North-ampt.	Chester	
		Males	Fem.					Males	Fem.
0	1000	1000	1000	1000	1000	1000	1000	1000	1000
1	680	577	611	542	633	730	743	773	828
2	548	497	521	471	528	595	625	678	739
3	492	443	468	430	485	544	582	624	683
4	452	405	433	400	434	517	553	588	648
5	426	381	413	377	403	498	536	571	623
6	410	366	398	357	387	481	521	556	609
7	397	355	386	344	376	467	509	543	601
8	388	346	377	337	367	457	499	537	596
9	380	338	370	331	361	447	492	528	591
10	373	330	364	327	356	440	487	524	588
11	367	325	359	322	353	434	483	521	585
12	361	321	356	318	350	429	478	519	583
13	356	317	354	314	347	424	474	516	579
14	351	314	351	310	344	419	470	513	576
15	347	310	348	306	341	415	465	511	572
16	343	306	345	302	338	411	461	506	568
17	338	301	342	299	335	407	457	501	563
18	334	296	339	295	332	403	452	496	558
19	329	291	335	291	328	398	446	490	553
20	325	285	331	288	324	394	441	485	547
21	321	279	327	284	320	389	434	479	543
22	316	273	323	280	315	384	428	473	538
23	310	267	319	276	310	379	421	467	533
24	305	261	315	273	305	374	415	461	528
25	299	254	311	269	297	369	409	454	523
26	294	247	306	265	293	364	402	448	515
27	288	240	302	261	287	358	396	441	508
28	283	233	297	256	281	352	389	434	500
29	278	226	291	251	275	346	383	428	493
30	272	219	285	247	269	341	376	422	485
31	266	212	279	243	264	336	370	417	479
32	260	205	273	239	259	331	364	412	473
33	254	198	267	235	254	326	357	407	467
34	248	192	261	231	249	321	351	402	461
35	242	185	254	226	243	316	344	397	455

TABLE I.

several Ages, and Places therein mentioned.

Ages	Breslaw	Sweden			Parish in Brand.	Holy Cross	Vaud	Annts. in Holland	Tonts. in France
		Males	Fem.	Both					
0	1000	1000	1000	1000	1000	1000	1000	1000	1000
1	769	770	791	780	775	817	811	804	745
2	658	720	739	730	718	754	765	768	709
3	614	686	704	695	687	708	735	730	682
4	585	662	679	671	664	677	715	709	662
5	563	647	666	656	642	651	701	689	647
6	546	635	654	644	622	630	688	676	634
7	532	624	643	634	607	614	677	664	624
8	523	615	635	625	595	601	667	652	615
9	515	608	628	618	585	594	659	646	607
10	508	601	622	611	577	589	653	639	600
11	502	596	616	606	570	585	648	633	595
12	497	591	612	602	564	582	643	627	590
13	492	587	608	597	559	579	639	621	585
14	488	583	604	594	554	576	635	616	581
15	483	579	601	590	549	573	631	611	578
16	479	575	597	586	544	569	626	606	574
17	474	571	593	582	539	565	622	601	570
18	470	567	589	578	535	561	618	596	565
19	465	563	585	574	531	557	614	590	561
20	461	558	581	570	527	553	610	584	556
21	456	553	577	565	522	548	606	577	551
22	451	548	572	560	517	542	602	571	545
23	446	543	568	555	512	537	597	566	540
24	441	538	564	551	507	531	592	559	534
25	436	532	559	546	502	525	587	551	529
26	431	527	555	541	498	519	582	543	523
27	426	521	550	535	495	512	577	535	517
28	421	516	544	530	492	506	572	526	512
29	415	510	539	525	489	500	567	517	506
30	409	505	533	519	486	494	563	508	500
31	403	499	527	513	482	489	558	499	495
32	397	493	521	507	477	483	553	490	490
33	391	487	515	501	472	478	548	482	484
34	384	481	508	495	467	472	544	474	479
35	377	475	502	488	462	466	539	467	474

TABLE I Continued.

Ages	Lond.	Stockholm		Vienna	Berlin	Norwich	Northampt.	Chester	
		Males	Fem.					Males	Fem.
36	236	179	248	221	237	311	338	391	448
37	230	172	242	216	230	306	331	385	442
38	224	166	236	211	223	300	325	379	435
39	218	160	230	205	216	295	318	373	429
40	212	154	223	199	209	290	312	366	422
41	207	148	217	194	203	284	305	360	415
42	201	142	210	189	197	279	299	352	408
43	194	136	204	185	192	274	292	345	401
44	187	130	197	181	187	268	285	337	394
45	180	124	190	176	182	263	279	329	387
46	174	119	184	171	177	257	272	322	380
47	167	113	177	165	172	251	265	314	373
48	159	108	171	159	167	245	259	306	366
49	153	102	164	153	162	239	252	298	359
50	147	97	158	147	157	233	245	290	352
51	141	91	152	142	152	227	238	281	345
52	135	86	146	137	147	221	231	273	338
53	130	81	141	133	142	215	224	264	331
54	125	76	135	128	137	208	217	256	325
55	120	71	130	123	132	202	210	249	318
56	116	67	125	117	127	195	203	241	312
57	111	62	120	111	121	188	196	234	306
58	106	58	115	106	115	181	189	226	300
59	101	54	110	101	109	175	182	219	293
60	96	50	105	96	103	168	175	211	286
61	92	46	100	91	97	161	168	201	277
62	87	42	94	87	92	154	161	190	265
63	83	38	88	82	88	147	154	178	253
64	78	35	82	77	84	139	147	167	242
65	74	31	77	72	80	132	140	156	232
66	70	28	72	67	75	124	133	148	224
67	65	26	67	62	70	116	126	141	217
68	61	23	62	57	65	109	119	136	210
69	56	21	57	52	60	101	113	130	202
70	52	20	52	48	55	94	106	123	193
71	47	17	47	44	51	86	99	115	181
72	43	15	42	40	47	79	92	103	167
73	39	13	38	36	43	73	85	92	153
74	35	11	33	33	39	66	78	81	139
75	32	10	28	30	35	59	71	72	127

TABLE I Continued.

Ages	Breslaw	Sweden			Parish in Brand.	Holy Cross	Vaud	Annts. in Holland	Tonts. in France
		Males	Fem.	Both					
36	370	469	496	482	436	460	533	460	460
37	363	463	490	477	450	453	527	453	464
38	356	457	485	471	444	447	520	446	459
39	349	451	479	465	438	441	513	439	454
40	342	445	473	459	432	435	506	432	449
41	335	438	467	453	427	428	500	425	444
42	328	431	459	445	422	422	494	419	439
43	321	423	452	437	417	416	488	413	434
44	314	415	444	430	412	409	482	407	429
45	307	407	437	422	407	402	476	400	424
46	299	399	429	414	400	394	469	393	419
47	291	391	423	407	394	387	461	386	413
48	283	383	416	400	388	380	451	378	408
49	275	375	410	392	381	373	441	370	402
50	267	367	403	385	374	365	431	362	396
51	259	357	395	376	367	358	422	354	390
52	250	348	387	367	359	351	414	345	384
53	241	338	379	358	351	344	406	336	378
54	232	329	370	349	343	335	397	327	371
55	224	319	362	340	334	327	388	318	363
56	216	310	353	331	324	319	377	309	355
57	209	300	345	322	314	310	364	300	346
58	201	290	336	312	304	301	348	291	338
59	193	280	327	303	293	292	331	282	329
60	186	270	317	293	282	283	314	273	319
61	178	260	306	282	271	273	299	264	309
62	170	249	294	271	260	264	286	255	299
63	163	237	282	259	248	255	274	245	288
64	155	226	270	247	236	245	262	235	278
65	147	214	258	235	224	236	250	225	267
66	140	203	246	224	213	226	236	215	256
67	132	191	234	212	202	216	220	205	245
68	124	179	222	200	190	206	202	195	234
69	117	167	210	187	178	195	184	185	222
70	109	154	198	175	166	185	168	175	211
71	101	142	185	162	153	175	153	165	199
72	93	129	171	149	138	164	140	155	187
73	85	117	156	135	122	155	129	145	175
74	77	105	140	121	107	144	119	135	162
75	69	94	125	108	93	134	109	125	148

TABLE I Continued.

Ages	Lond.	Stockholm		Vienna	Berlin	Norwich	Northampt.	Chester	
		Males	Fem.					Males	Fem.
76	28	8	23	27	32	52	65	64	116
77	25	6	19	24	29	46	58	58	106
78	22	5	16	21	26	40	52	52	96
79	19	4	13	18	23	35	46	47	86
80	17	3	10	16	20	31	40	41	76
81	14	2	8	14	18	27	35	36	66
82	12	2	6	12	16	24	30	31	57
83	10	1	4	10	14	20	25	26	47
84	8	1	3	8	12	17	20	22	37
85	7	1	2	7	10	14	16	19	29
86	6	0	1	6	8	12	12	15	23
87	5	0	1	5	7	9	9	13	19
88	4	0	0	4	6	8	7	11	16
89	3	0	0	3	5	6	5	9	14
90	2	0	0	2	4	4	4	7	13
91	1	0	0	1	3	2	3	6	11
92	0	0	0	0	2	1	2	4	9
93	0	0	0	0	1	0	1	2	7
94	0	0	0	0	0	0	1	1	5
95	0	0	0	0	0	0	0	0	3
96	0	0	0	0	0	0	0	0	2
97	0	0	0	0	0	0	0	0	1

TABLE I Continued.

Ages	Breslaw	Sweden			Parish in Brand	Holy Cross	Vaud	Annts. in Holland	Tonts. in France
		Males	Fem.	Both					
76	61	84	111	96	80	124	98	114	134
77	53	74	98	84	68	115	85	103	120
78	45	65	86	75	59	106	71	92	106
79	38	56	75	65	51	96	58	82	94
80	32	48	65	56	44	87	46	72	81
81	26	41	55	47	38	78	36	62	70
82	22	34	46	38	32	69	29	53	59
83	18	27	38	31	25	61	24	45	49
84	15	21	30	24	21	53	20	38	40
85	12	16	22	19	15	44	17	31	33
86	9	12	17	14	11	36	14	25	26
87	6	9	13	11	8	29	11	19	21
88	4	7	10	8	6	23	9	14	16
89	2	5	8	6	4	18	7	10	12
90	1	4	6	5	3	13	5	7	8
91	0	3	4	3	2	10	4	5	5
92	0	2	3	2	1	8	3	4	3
93	0	1	2	1	0	6	2	2	1
94	0	0	1	0	0	4	1	1	1
95	0	0	1	0	0	2	0	0	0
96	0	0	0	0	0	1	0	0	0
97	0	0	0	0	0	0	0	0	0

TABLE II.

Showing the *Expectation of Life*, at the

Ages	Lond.	Stockholm		Vienna	Berlin	Norwich	Northampt.	Chester	
		Males	Fem.					Males	Fem.
Birth.	17·90	14·25	18·10	16·37	17·85	23·16	25·18	28·13	33·27
5	35·28	31·05	37·12	36·54	28·67	40·22	40·84	43·20	47·44
10	34·91	30·00	36·89	37·02	37·15	40·26	39·78	41·92	45·17
15	32·32	26·74	33·43	34·11	33·65	37·50	36·51	38·05	41·36
20	29·37	23·85	30·01	31·39	30·34	34·37	33·43	34·86	38·10
25	26·66	21·40	26·80	28·32	27·47	31·56	30·85	32·00	34·78
30	24·11	19·42	23·98	25·62	25·25	28·93	28·27	29·25	32·27
35	21·76	17·58	21·62	22·66	22·76	26·06	25·68	25·97	29·26
40	19·50	15·61	19·25	20·49	20·91	23·18	23·08	22·92	26·37
45	17·63	13·78	17·17	17·82	18·85	20·30	20·52	20·20	23·50
50	15·84	11·95	15·12	15·88	16·40	17·55	17·99	17·64	20·62
55	13·91	10·30	12·89	13·50	14·14	14·88	15·58	15·14	17·52
60	11·69	8·69	10·45	11·65	12·49	12·36	13·21	12·36	14·20
65	9·69	7·39	8·39	9·51	10·48	10·06	10·88	10·79	11·94
70	8·00	5·81	6·16	8·30	8·69	8·12	8·60	8·05	8·81
75	6·27	4·09	4·39	6·37	7·08	6·44	6·54	7·00	7·14
80	4·86	3·20	3·09	5·50	6·07	5·15	4·75	5·43	5·20
85	3·04	1·78	2·02	3·33	4·50	3·50	3·37	4·25	4·81
90	·00	·00	·00	1·50	2·83	1·50	2·41	2·50	3·46
95	·00	·00	·00	·00	·00	·00	·75	·00	1·21

TABLE II.

several Ages, and Places therein mentioned.

Age	Breslaw	Sweden			Parish in Brand.	Holy Cross	Vaud	Annts in dollars.	Tonts. in France
		Males	Fem.	Both					
Birth.	26·60	33·20	35·70	34·42	32·51	33·93	37·56	34·47	34·79
5	41·47	45·62	48·00	46·7	45·08	46·30	47·11	44·45	48·19
10	40·40	43·94	46·25	45·07	44·93	46·00	45·66	42·71	46·76
15	37·40	40·56	42·76	41·64	42·10	42·25	42·17	39·55	43·46
20	34·15	36·95	39·15	38·02	38·76	38·66	38·53	36·31	40·08
25	30·89	33·63	35·58	34·58	35·56	35·58	34·84	33·27	37·01
30	27·81	30·34	32·17	31·21	31·66	32·66	31·33	30·92	33·96
35	24·92	27·09	29·03	28·03	28·17	29·43	27·61	28·36	30·73
40	22·19	23·75	25·21	24·66	24·95	26·40	24·24	25·49	27·30
45	19·56	20·71	22·57	21·61	21·33	23·35	20·62	22·34	23·77
50	17·07	17·72	19·26	18·46	17·98	20·40	17·49	19·41	20·24
55	14·77	14·98	16·15	15·53	14·83	17·47	14·14	16·72	16·88
60	12·31	12·24	13·08	12·63	12·09	14·85	11·84	14·10	13·86
65	9·86	9·78	10·49	10·10	9·57	12·30	9·26	11·56	11·07
70	7·45	7·60	7·91	7·72	7·01	10·00	7·52	9·15	8·34
75	5·51	5·89	6·03	5·91	5·53	7·87	5·36	6·81	5·79
80	4·07	4·27	4·47	4·28	4·27	5·75	4·24	5·05	4·73
85	2·37	3·16	3·40	3·23	2·83	3·90	3·21	3·38	3·45
90	·00	2·02	2·55	2·05	1·50	2·89	·00	2·47	1·79
95	·00	·00	1·07	1·00	·00	1·00	·00	·00	·83

TABLE III.

Showing the Number of Persons *living* and *dying* at every age, according to the observations of *M. De Parcieux.*

Age	Living	Dying	Age	Living	Dying	Age	Living	Dying
3	1000	30	34	702	8	65	395	15
4	970	22	35	694	8	66	380	16
5	948	18	36	686	8	67	364	17
6	930	15	37	678	7	68	347	18
7	915	13	38	671	7	69	329	19
8	902	12	39	664	7	70	310	19
9	890	10	40	657	7	71	291	20
10	880	8	41	650	7	72	271	20
11	872	6	42	643	7	73	251	20
12	866	6	43	636	7	74	231	20
13	860	6	44	629	7	75	211	19
14	854	6	45	622	7	76	192	19
15	848	6	46	615	8	77	173	19
16	842	7	47	607	8	78	154	18
17	835	7	48	599	9	79	136	18
18	828	7	49	590	9	80	118	17
19	821	7	50	581	10	81	101	16
20	814	8	51	571	11	82	85	14
21	806	8	52	560	11	83	71	12
22	798	8	53	549	11	84	59	11
23	790	8	54	538	12	85	48	10
24	782	8	55	526	12	86	38	9
25	774	8	56	514	12	87	29	7
26	766	8	57	502	13	88	22	6
27	758	8	58	489	13	89	16	5
28	750	8	59	476	13	90	11	4
29	742	8	60	463	13	91	7	3
30	734	8	61	450	13	92	4	2
31	726	8	62	437	14	93	2	1
32	718	8	63	423	14	94	1	1
33	710	8	64	409	14	95	0	0

TABLE IV.

Showing the *Expectation of Life*, according to the observations of *M. De Parcieux*.

Age	Expectation	Age	Expectation	Age	Expectation
3	47·71	34	31·52	65	11·26
4	48·17	35	30·88	66	10·69
5	48·27	36	30·23	67	10·14
6	48·20	37	29·58	68	9·61
7	47·98	38	28·89	69	9·11
8	47·66	39	28·18	70	8·64
9	47·30	40	27·48	71	8·17
10	46·83	41	26·77	72	7·73
11	46·26	42	26·06	73	7·31
12	45·58	43	25·34	74	6·90
13	44·89	44	24·62	75	6·50
14	44·20	45	23·89	76	6·10
15	43·51	46	23·15	77	5·71
16	42·82	47	22·45	78	5·36
17	42·17	48	21·74	79	5·00
18	41·52	49	21·07	80	4·69
19	40·87	50	20·38	81	4·39
20	40·22	51	19·73	82	4·01
21	39·62	52	19·11	83	3·84
22	39·00	53	18·48	84	3·52
23	38·40	54	17·85	85	3·21
24	37·78	55	17·25	86	2·92
25	37·17	56	16·64	87	2·67
26	36·55	57	16·02	88	2·36
27	35·93	58	15·44	89	2·06
28	35·30	59	14·84	90	1·77
29	34·69	60	14·25	91	1·50
30	34·06	61	13·65	92	1·25
31	33·29	62	13·04	93	1·00
32	32·80	63	12·43	94	·00
33	32·16	64	11·86		

TABLE V.

Showing the value of an annuity on a Single Life, deduced from the observations of *M. De Parcieux.*

Ages	3 per Cent	$3\frac{1}{2}$ per Cent	4 per Cent	$4\frac{1}{2}$ per Cent	5 per Cent	6 per Cent
3	22·028	19·980	18·242	16·756	15·475	13·391
4	22·390	20·319	18·559	17·052	15·752	13·633
5	22·597	20·518	18·749	17·233	15·923	13·787
6	22·726	20·647	18·877	17·357	16·043	13·897
7	22·791	20·720	18·953	17·435	16·121	13·972
8	22·813	20·754	18·996	17·482	16·171	14·024
9	22·815	20·770	19·022	17·515	16·209	14·066
10	22·766	20·742	19·008	17·512	16·213	14·079
11	22·664	20·665	18·949	17·468	16·179	14·061
12	22·506	20·556	18·844	17·380	16·106	14·007
13	22·343	20·403	18·734	17·289	16·029	13·951
14	22·175	20·266	18·620	17·194	15·949	13·892
15	22·002	20·123	18·502	17·095	15·865	13·830
16	21·823	19·976	18·380	16·991	15·777	13·765
17	21·666	19·849	18·275	16·905	15·705	13·713
18	21·505	19·717	18·167	16·815	15·629	13·658
19	21·339	19·581	18·054	16·721	15·551	13·601
20	21·168	19·441	17·938	16·624	15·469	13·541
21	21·020	19·321	17·841	16·544	15·402	13·496
22	20·867	19·197	17·740	16·462	15·336	13·450
23	20·711	19·071	17·637	16·377	15·265	13·401
24	20·550	18·940	17·530	16·289	15·193	13·350
25	20·386	18·805	17·420	16·198	15·117	13·298
26	20·217	18·667	17·306	16·104	15·039	13·243
27	20·043	18·524	17·188	16·006	14·957	13·185
28	19·864	18·377	17·066	15·905	14·873	13·126
29	19·681	18·225	16·940	15·800	14·785	13·063
30	19·492	18·069	16·810	15·691	14·693	12·998
31	19·298	17·907	16·675	15·578	14·598	12·930
32	19·099	17·741	16·535	15·460	14·499	12·858
33	18·893	17·568	16·390	15·338	14·395	12·783
34	18·682	17·390	16·240	15·211	14·287	12·704
35	18·464	17·207	16·084	15·078	14·175	12·622

TABLE V Continued.

Age.	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
36	18·240	17·017	15·922	14·941	14·057	12·535
37	18·009	16·820	15·755	14·797	13·934	12·444
38	17·742	16·590	15·556	14·624	13·783	12·329
39	17·467	16·352	15·349	14·444	13·625	12·206
40	17·183	16·105	15·133	14·254	13·459	12·076
41	16·889	15·848	14·907	14·056	13·284	11·939
42	16·585	15·581	14·673	13·849	13·100	11·793
43	16·271	15·304	14·427	13·631	12·906	11·638
44	15·946	15·016	14·171	13·403	12·702	11·473
45	15·609	14·716	13·904	13·164	12·487	11·299
46	15·260	14·405	13·625	12·913	12·261	11·113
47	14·925	14·105	13·357	12·672	12·044	10·935
48	14·578	13·794	13·076	12·419	11·815	10·746
49	14·244	13·494	12·807	12·176	11·595	10·564
50	13·899	13·1[illegible]3	12·526	11·921	11·363	10·372
51	13·567	12·883	12·255	11·675	11·140	10·187
52	13·248	12·596	11·995	11·440	10·927	10·010
53	12·919	12·298	11·725	11·195	10·703	9·823
54	12·579	11·989	11·443	10·938	10·468	9·625
55	12·252	11·692	11·173	10·691	10·242	9·436
56	11·914	11·383	10·891	10·433	10·006	9·235
57	11·565	11·063	10·597	10·163	9·757	9·024
58	11·228	10·755	10·314	9·902	9·517	8·819
59	10·881	10·436	10·020	9·631	9·266	8·604
60	10·522	10·104	9·713	9·346	9·003	8·376
61	10·151	9·760	9·393	9·049	8·726	8·135
62	9·766	9·402	9·060	8·738	8·435	7·880
63	9·392	9·053	8·734	8·433	8·150	7·629
64	9·005	8·691	8·394	8·114	7·850	7·363
65	8·604	8·314	8·039	7·780	7·535	7·082
66	8·212	7·944	7·691	7·451	7·224	6·803
67	7·830	7·584	7·350	7·129	6·918	6·528
68	7·460	7·234	7·019	6·814	6·620	6·259
69	7·104	6·896	6·699	6·511	6·331	5·997
70	6·766	6·575	6·394	6·221	6·055	5·747
71	6·424	6·250	6·084	5·925	5·773	5·489
72	6·105	5·946	5·794	5·648	5·505	5·248
73	5·789	5·644	5·506	5·373	5·246	5·006
74	5·479	5·348	5·222	5·101	4·985	4·766
75	5·178	5·060	4·945	4·836	4·730	4·531

TABLE V Continued.

Ages	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
76	4·862	4·755	4·652	4·553	4·458	4·278
77	4·557	4·462	4·370	4·281	4·195	4·033
78	4·273	4·188	4·105	4·025	3·948	3·802
79	3·984	3·908	3·834	3·763	3·694	3·563
80	3·730	3·662	3·596	3·533	3·471	3·353
81	3·448	3·428	3·370	3·313	3·258	3·153
82	3·269	3·216	3·164	3·114	3·065	2·971
83	3·031	2·984	2·939	2·895	2·853	2·770
84	2·757	2·717	2·679	2·641	2·604	2·534
85	2·490	2·457	2·424	2·392	2·361	2·301
86	2·240	2·212	2·185	2·158	2·132	2·081
87	2·023	2·000	1·977	1·955	1·933	1·891
88	1·747	1·728	1·711	1·693	1·675	1·642
89	1·474	1·460	1·446	1·433	1·418	1·394
90	1·208	1·198	1·187	1·178	1·166	1·149
91	·955	·948	·941	·934	·924	·913
92	·721	·716	·712	·707	·703	·694
93	·485	·483	·481	·478	·476	·472
94	·000	·000	·000	·000	·000	·000

TABLE VI.

Showing the value of an annuity on Two Joint Lives, deduced from the observations of *M. De Parcieux.*

Difference of age 0 years.

Common Age	3½ per Cent	4½ per Cent	Common Age	3½ per Cent	4½ per Cent	Common Age	3½ per Cent	4½ per Cent
			36	13·839	12·408	71	3·968	3·817
			37	13·663	12·274	72	3·735	3·599
3	15·785	13·571	38	13·438	12·095	73	3·507	3·384
4	16·364	14·072	39	13·203	11·907	74	3·285	3·176
5	16·732	14·396	40	12·958	11·710	75	3·075	2·977
6	16·994	14·632	41	12·702	11·502	76	2·844	2·758
7	17·171	14·796	42	12·434	11·282	77	2·625	2·549
8	17·287	14·910	43	12·154	11·051	78	2·429	2·362
9	17·378	15·004	44	11·861	10·807	79	2·224	2·165
10	17·398	15·038	45	11·554	10·549	80	2·057	2·005
11	17·338	15·004	46	11·232	10·276	81	1·906	1·860
12	17·195	14·897	47	10·934	10·023	82	1·786	1·745
13	17·046	14·786	48	10·621	9·756	83	1·649	1·614
14	16·891	14·669	49	10·331	9·508	84	1·472	1·442
15	16·731	14·547	50	10·026	9·246	85	1·301	1·276
16	16·564	14·419	51	9·743	9·004	86	1·149	1·128
17	16·432	14·321	52	9·485	8·782	87	1·042	1·024
18	16·296	14·220	53	9·214	8·549	88	·873	·860
19	16·155	14·114	54	8·930	8·303	89	·709	·699
20	16·010	14·004	55	8·669	8·077	90	·552	·545
21	15·901	13·926	56	8·397	7·839	91	·410	·405
22	15·789	13·846	57	8·111	7·588	92	·300	·296
23	15·674	13·764	58	7·847	7·357	93	·242	·239
24	15·556	13·679	59	7·572	7·114	94	·000	·000
25	15·435	13·591	60	7·283	6·857			
26	15·311	13·501	61	6·979	6·585			
27	15·183	13·408	62	6·660	6·297			
28	15·051	13·312	63	6·357	6·024			
29	14·916	13·213	64	6·037	5·733			
30	14·776	13·110	65	5·700	5·423			
31	14·632	13·004	66	5·374	5·123			
32	14·484	12·893	67	5·062	4·835			
33	14·331	12·779	68	4·765	4·560			
34	14·172	12·660	69	4·486	4·300			
35	14·008	12·536	70	4·230	4·062			

TABLE VII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations of *M. De Parcieux.* Difference of age 5 years.

Ages	$3\frac{1}{2}$ per Cent	$4\frac{1}{2}$ per Cent	Ages	$3\frac{1}{2}$ per Cent	$4\frac{1}{2}$ per Cent	Ages	$3\frac{1}{2}$ per Ct	$4\frac{1}{2}$ per Ct.
			31–36	14·199	12·680	61–66	6·065	5·757
			32–37	14·035	12·550	62–67	5·749	5·467
3– 8	16·510	14·219	33–38	13·843	12·408	63–68	5·448	5·191
4– 9	16·854	14·525	34–39	13·644	12·252	64–69	5·151	4·917
5–10	17·052	14·708	35–40	13·436	12·089	65–70	4·858	4·647
6–11	17·155	14·811	36–41	13·220	11·918	66–71	4·568	4·377
7–12	17·172	14·840	37–42	12·995	11·739	67–72	4·300	4·128
8–13	17·154	14·841	38–43	12·740	11·531	68–73	4·040	3·885
9–14	17·121	14·828	39–44	12·473	11·313	69–74	3·792	3·653
10–15	17·048	14·783	40–45	12·194	11·082	70–75	3·561	3·436
11–16	16·934	14·701	41–46	11·902	10·839	71–76	3·314	3·203
12–17	16·796	14·598	42–47	11·617	10·601	72–77	3·088	2·989
13–18	16·653	14·491	43–48	11·318	10·349	73–78	2·877	2·788
14–19	16·505	14·380	44–49	11·025	10·102	74–79	2·663	2·585
15–20	16·351	14·264	45–50	10·718	9·841	75–80	2·478	2·409
16–21	16·213	14·161	46–51	10·416	9·583	76–81	2·293	2·232
17–22	16·091	14·071	47–52	10·137	9·345	77–82	2·130	2·076
18–23	15·965	13·979	48–53	9·845	9·095	78–83	1·964	1·917
19–24	15·835	13·884	49–54	9·557	8·846	79–84	1·770	1·730
20–25	15·701	13·784	50–55	9·274	8·602	80–85	1·596	1·562
21–26	15·583	13·699	51–56	8·994	8·360	81–86	1·437	1·408
22–27	15·462	13·612	52–57	8·719	8·120	82–87	1·316	1·292
23–28	15·338	13·522	53–58	8·449	7·886	83–88	1·150	1·130
24–29	15·210	13·429	54–59	8·168	7·639	84–89	·969	·954
25–30	15·079	13·333	55–60	7·889	7·394	85–90	·793	·782
26–31	14·943	13·233	56–61	7·597	7·136	86–91	·630	·621
27–32	14·804	13·131	57–62	7·291	6·862	87–92	·495	·489
28–33	14·660	13·024	58–63	7·003	6·605	88–93	·351	·348
29–34	14·511	12·914	59–64	6·701	6·333	89–94	·000	·000
30–35	14·358	12·799	60–65	6·383	6·045			

TABLE VIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations of *M. De Parcieux.* Difference of age 10 years.

Ages	3½ per Cent	4½ per Cent	Ages	3½ per Cent	4½ per Cent	Ages	3½ per Ct.	4½ per Ct.
			31–41	13·500	12·133	61–71	5·054	4·828
			32–42	13·282	11·960	62–72	4·784	4·578
3–13	16·366	13·535	33–43	13·055	11·778	63–73	4·523	4·337
4–14	16·586	14·345	34–44	12·818	11·587	64–74	4·261	4·093
5–15	16·689	14·446	35–45	12·571	11·385	65–75	3·999	3·848
6–16	16·733	14·498	36–46	12·312	11·174	66–76	3·728	3·594
7–17	16·750	14·528	37–47	12·063	10·970	67–77	3·471	3·351
8–18	16·734	14·531	38–48	11·784	10·738	68–78	3·233	3·126
9–19	16·703	14·521	39–49	11·513	10·512	69–79	2·996	2·902
10–20	16·635	14·479	40–50	11·230	10·274	70–80	2·793	2·709
11–21	16·547	14·421	41–51	10·954	10·042	71–81	2·598	2·524
12–22	16·418	14·326	42–52	10·686	9·817	72–82	2·431	2·365
13–23	16·285	14·228	43–53	10·406	9·579	73–83	2·253	2·195
14–24	16·147	14·126	44–54	10·112	9·329	74–84	2·049	1·999
15–25	16·004	14·020	45–55	9·825	9·083	75–85	1·853	1·811
16–26	15·856	13·909	46–56	9·525	8·824	76–86	1·663	1·627
17–27	15·724	13·811	47–57	9·227	8·566	77–87	1·503	1·473
18–28	15·586	13·710	48–58	8·935	8·312	78–88	1·303	1·279
19–29	15·445	13·605	49–59	8·645	8·060	79–89	1·100	1·082
20–30	15·299	13·496	50–60	8·342	7·793	80–90	·909	·895
21–31	15·168	13·400	51–61	8·039	7·526	81–91	·727	·717
22–32	15·032	13·301	52–62	7·736	7·257	82–92	·566	·559
23–33	14·893	13·199	53–63	7·437	6·992	83–93	·401	·398
24–34	14·750	13·092	54–64	7·124	6·711	84–94	·000	·000
25–35	14·601	12·982	55–65	6·809	6·428			
26–36	14·448	12·868	56–66	6·496	6·145			
27–37	14·290	12·749	57–67	6·187	5·864			
28–38	14·104	12·606	58–68	5·895	5·598			
29–39	13·911	12·455	59–69	5·611	5·339			
30–40	13·710	12·298	60–70	5·337	5·088			

TABLE IX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations of *M. De Parcieux.* Difference of age 20 years.

Ages	3½ per Cent	4½ per Cent	Ages	3½ per Cent	4½ per Cent	Ages	3½ per Ct.	4½ per Ct.
			31–51	11·398	10·407	61–81	3·039	2·944
			32–52	11·163	10·212	62–82	2·849	2·764
3–23	15·560	13·566	33–53	10·918	10·008	63–83	2·647	2·573
4–24	15·773	13·765	34–54	10·663	9·794	64–84	2·409	2·346
5–25	15·876	13·870	35–55	10·418	9·589	65–85	2·173	2·120
6–26	15·925	13·929	36–56	10·163	9·374	66–86	1·954	1·909
7–27	15·929	13·951	37–57	9·897	9·148	67–87	1·766	1·729
8–28	15·903	13·946	38–58	9·625	8·916	68–88	1·528	1·498
9–29	15·861	13·929	39–59	9·342	8·673	69–89	1·293	1·270
10–30	15·784	13 882	40–60	9·046	8·417	70–90	1·066	1·049
11–31	15·668	13·801	41–61	8·737	8·147	71–91	·846	·834
12–32	15·511	13·684	42–62	8·413	7·862	72–92	·646	·638
13–33	15·348	13·562	43–63	8·095	7·582	73–93	·445	·440
14–34	15·179	13·434	44–64	7·762	7·285	74–94	·000	·000
15–35	15·004	13·301	45–65	7·412	6·972			
16–36	14·822	13·162	46–66	7·065	6·659			
17–37	14·652	13·033	47–67	6·734	6·360			
18–38	14·452	12·878	48–68	6·409	6·065			
19–39	14·245	12·716	49–69	6·103	5·787			
20–40	14·028	12·545	50–70	5·807	5·517			
21–41	13 821	12·382	51–71	5·515	5·250			
22–42	13·606	12·211	52–72	5·250	5·007			
23–43	13·381	12·032	53–73	4·984	4·762			
24–44	13·147	11·843	54–74	4·720	4·518			
25–45	12·902	11·645	55–75	4·470	4·286			
26–46	12·647	11·436	56–76	4·203	4·037			
27–47	12·402	11·236	57–77	3·943	3·794			
28–48	12·146	11·025	58–78	3·706	3 573			
29–49	11·901	10·823	59–79	3·463	3·343			
30–50	11·645	10·611	60–80	3·246	3·140			

TABLE X.

Showing the value of an annuity on Two Joint Lives, deduced from the observations of *M. De Parcieux*. Difference of age 30 years.

Ages	$3\frac{1}{2}$ per Cent	$4\frac{1}{2}$ per Cent	Ages	$3\frac{1}{2}$ per Cent.	$4\frac{1}{2}$ per Cent
3–33	14·605	12·893	36–66	7·357	6·920
4–34	14·761	13·048	37–67	7·043	6·639
5–35	14·813	13·112	38–68	6·726	6·353
6–36	14·810	13·130	39–69	6·420	6·076
7–37	14·764	13·111	40–70	6·127	5·811
8–38	14·662	13·043	41–71	5·828	5·538
9–39	14·542	12·960	42–72	5·548	5·282
10–40	14·384	12·843	43–73	5·268	5·025
11–41	14·186	12·690	44–74	4·990	4·770
12–42	13·946	12·498	45–75	4·718	4·518
13–43	13·694	12·296	46–76	4·427	4·248
14–44	13·432	12·084	47–77	4·152	3·991
15–45	13·158	11·860	48–78	3·892	3·748
16–46	12·872	11·624	49–79	3·632	3·503
17–47	12·611	11·410	50–80	3·399	3·284
18–48	12·339	11·185	51–81	3·182	3·080
19–49	12·076	10·968	52–82	2·991	2·899
20–50	11·801	10·739	53–83	2·780	2·700
21–51	11·551	10·532	54–84	2·533	2·464
22–52	11·313	10·335	55–85	2·296	2·238
23–53	11·064	10·128	56–86	2·072	2·023
24–54	10·805	9·911	57–87	1·877	1·836
25–55	10·557	9·702	58–88	1·629	1·596
26–56	10·298	9·484	59–89	1·381	1·356
27–57	10·029	9·255	60–90	1·138	1·119
28–58	9·769	9·034	61–91	·904	·891
29–59	9·499	8·803	62–92	·686	·677
30–60	9·218	8·561	63–93	·467	·463
31–61	8·924	8·306	64–94	·000	·000
32–62	8·617	8·038			
33–63	8·318	7·775			
34–64	8·005	7·499			
35–65	7·678	7·207			

TABLE III.

Showing the Number of Persons *living* and *dying* at every age, according to the observations of *M. De Parcieux.*

Age	Living	Dying	Age	Living	Dying	Age	Living	Dying
3	1000	30	34	702	8	65	395	15
4	970	22	35	694	8	66	380	16
5	948	18	36	686	8	67	364	17
6	930	15	37	678	7	68	347	18
7	915	13	38	671	7	69	329	19
8	902	12	39	664	7	70	310	19
9	890	10	40	657	7	71	291	20
10	880	8	41	650	7	72	271	20
11	872	6	42	643	7	73	251	20
12	866	6	43	636	7	74	231	20
13	860	6	44	629	7	75	211	19
14	854	6	45	622	7	76	192	19
15	848	6	46	615	8	77	173	19
16	842	7	47	607	8	78	154	18
17	835	7	48	599	9	79	136	18
18	828	7	49	590	9	80	118	17
19	821	7	50	581	10	81	101	16
20	814	8	51	571	11	82	85	14
21	806	8	52	560	11	83	71	12
22	798	8	53	549	11	84	59	11
23	790	8	54	538	12	85	48	10
24	782	8	55	526	12	86	38	9
25	774	8	56	514	12	87	29	7
26	766	8	57	502	13	88	22	6
27	758	8	58	489	13	89	16	5
28	750	8	59	476	13	90	11	4
29	742	8	60	463	13	91	7	3
30	734	8	61	450	13	92	4	2
31	726	8	62	437	14	93	2	1
32	718	8	63	423	14	94	1	1
33	710	8	64	409	14	95	0	0

TABLE IV.

Showing the *Expectation of Life*, according to the observations of *M. De Parcieux*.

Age	Expectation	Age	Expectation	Age	Expectation
3	47·71	34	31·52	65	11·26
4	48·17	35	30·88	66	10·69
5	48·27	36	30·23	67	10·14
6	48·20	37	29·58	68	9·61
7	47·98	38	28·89	69	9·11
8	47·66	39	28·18	70	8·64
9	47·30	40	27·48	71	8·17
10	46·83	41	26·77	72	7·73
11	46·26	42	26·06	73	7·31
12	45·58	43	25·34	74	6·90
13	44·89	44	24·62	75	6·50
14	44·20	45	23·89	76	6·10
15	43·51	46	23·15	77	5·71
16	42·82	47	22·45	78	5·36
17	42·17	48	21·74	79	5·00
18	41·52	49	21·07	80	4·69
19	40·87	50	20·38	81	4·39
20	40·22	51	19·73	82	4·01
21	39·62	52	19·11	83	3·84
22	39·00	53	18·48	84	3·52
23	38·40	54	17·85	85	3·21
24	37·78	55	17·25	86	2·92
25	37·17	56	16·64	87	2·67
26	36·55	57	16·02	88	2·36
27	35·93	58	15·44	89	2·06
28	35·30	59	14·84	90	1·77
29	34·69	60	14·25	91	1·50
30	34·06	61	13·65	92	1·25
31	33·29	62	13·04	93	1·00
32	32·80	63	12·43	94	·00
33	32·16	64	11·86		

TABLE V.

Showing the value of an annuity on a Single Life, deduced from the observations of *M. De Parcieux.*

Ages	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
3	22·028	19·980	18·242	16·756	15·475	13·391
4	22·390	20·319	18·559	17·052	15·752	13·633
5	22·597	20·518	18·749	17·253	15·923	13·787
6	22·726	20·647	18·877	17·357	16·043	13·897
7	22·791	20·720	18·953	17·435	16·121	13·972
8	22·813	20·754	18·996	17·482	16·171	14·024
9	22·815	20·770	19·022	17·5[illegible]5	16·209	14·066
10	22·766	20·742	19·008	17·512	16·213	14·079
11	22·664	20·665	18·949	17·468	16·179	14·061
12	22·500	20·550	18·844	17·380	16·106	14·007
13	22·343	20·403	18·734	17·289	16·020	13·951
14	22·175	20·266	18·620	17·194	15·940	13·892
15	22·002	20·123	18·502	17·095	15·865	13·830
16	21·823	19·976	18·380	16·991	15·777	13·765
17	21·666	19·840	18·275	16·905	15·705	13·713
18	21·505	19·717	18·167	16·815	15·629	13·658
19	21·339	19·581	18·054	16·721	15·551	13·601
20	21·168	19·441	17·938	16·624	15·469	13·541
21	21·020	19·321	17·841	16·544	15·403	13·496
22	20·867	19·197	17·740	16·462	15·336	13·450
23	20·711	19·071	17·637	16·377	15·265	13·401
24	20·550	18·940	17·530	16·289	15·193	13·350
25	20·385	18·805	17·420	16·19[illegible]	15·117	13·298
26	20·217	18·667	17·306	16·104	15·039	13·243
27	20·043	18·524	17·188	16·000	14·957	13·185
28	19·864	18·377	17·066	15·905	14·873	13·126
29	19·681	18·225	16·940	15·800	14·785	13·065
30	19·492	18·069	16·810	15·691	14·693	12·996
31	19·298	17·907	16·675	15·578	14·598	12·930
32	19·099	17·741	16·535	15·460	14·499	12·858
33	18·893	17·568	16·390	15·338	14·395	12·783
34	18·682	17·390	16·240	15·211	14·287	12·704
35	18·464	17·207	16·084	15·078	14·175	12·622

TABLE XIV.

owing the Number of Persons *living* and *dying* at every age, according to the observations made in *Sweden*.

	Males		Females		Lives in general	
Ages	Living	Dying	Living	Dying	Living	Dying
0	10000	2300	10000	2090	10000	2195
1	7700	500	7910	518	7805	509
2	7200	337	7392	350	7296	344
3	6863	240	7042	250	6952	245
4	6623	150	6792	135	6707	143
5	6473	125	6657	120	6564	122
6	6348	105	6537	105	6442	105
7	6243	90	6432	85	6337	87
8	6153	75	6347	70	6250	73
9	6078	65	6277	60	6177	62
10	6013	55	6217	52	6115	54
11	5958	45	6165	46	6061	45
12	5913	45	6119	40	6016	42
13	5868	40	6079	35	5974	38
14	5828	40	6044	35	5936	37
15	5788	39	6009	35	5899	37
16	5749	39	5974	40	5862	40
17	5710	39	5934	40	5822	40
18	5671	44	5894	42	5782	42
19	5627	44	5852	43	5740	43
20	5583	50	5809	43	5697	47
21	5533	50	5766	43	5650	47
22	5483	50	5723	43	5603	48
23	5433	55	5680	44	5555	48
24	5378	55	5636	45	5507	50
25	5323	55	5591	45	5457	50
26	5268	55	5546	50	5407	52
27	5213	55	5496	52	5355	54
28	5158	55	5444	55	5301	55
29	5103	56	5389	55	5246	55
30	5049	59	5334	60	5191	59

TABLE V.

Showing the value of an annuity on a Single Life, deduced from the observations of *M. De Parcieux.*

Ages	3 per Cent	$3\frac{1}{2}$ per Cent	4 per Cent	$4\frac{1}{2}$ per Cent	5 per Cent	6 per Cent
3	22·028	19·980	18·242	16·756	15·475	13·391
4	22·390	20·319	18·559	17·052	15·752	13·633
5	22·597	20·518	18·749	17·253	15·923	13·787
6	22·726	20·647	18·877	17·357	16·043	13·897
7	22·791	20·720	18·953	17·435	16·121	13·972
8	22·813	20·754	18·996	17·482	16·171	14·024
9	22·815	20·770	19·022	17·5[illegible]5	16·209	14·066
10	22·766	20·742	19·008	17·512	16·213	14·079
11	22·664	20·665	18·949	17·468	16·179	14·061
12	22·500	20·550	18·844	17·380	16·106	14·007
13	22·343	20·403	18·734	17·289	16·029	13·951
14	22·175	20·266	18·620	17·194	15·949	13·892
15	22·002	20·123	18·502	17·095	15·865	13·830
16	21·823	19·976	18·380	16·991	15·777	13·765
17	21·666	19·849	18·275	16·905	15·705	13·713
18	21·505	19·717	18·167	16·815	15·629	13·658
19	21·339	19·581	18·054	16·721	15·551	13·601
20	21·168	19·441	17·938	16·624	15·469	13·541
21	21·020	19·321	17·841	16·544	15·403	13·496
22	20·867	19·197	17·740	16·462	15·336	13·450
23	20·711	19·071	17·637	16·377	15·265	13·401
24	20·550	18·940	17·530	16·289	15·193	13·350
25	20·386	18·805	17·420	16·198	15·117	13·298
26	20·217	18·667	17·306	16·104	15·039	13·243
27	20·043	18·524	17·188	16·006	14·957	13·185
28	19·864	18·377	17·066	15·905	14·873	13·126
29	19·681	18·225	16·940	15·800	14·785	13·063
30	19·492	18·069	16·810	15·691	14·693	12·998
31	19·298	17·907	16·675	15·578	14·598	12·930
32	19·099	17·741	16·535	15·460	14·499	12·858
33	18·893	17·568	16·390	15·338	14·395	12·783
34	18·682	17·390	16·240	15·211	14·287	12·704
35	18·464	17·207	16·084	15·078	14·175	12·622

TABLE V Continued.

Age.	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
36	18·240	17·017	15·922	14·941	14·057	12·535
37	18·009	16·820	15·755	14·797	13·934	12·444
38	17·742	16·590	15·556	14·624	13·783	12·329
39	17·467	16·352	15·349	14·444	13·625	12·206
40	17·183	16·105	15·133	14·254	13·459	12·076
41	16·889	15·848	14·907	14·056	13·284	11·939
42	16·585	15·581	14·673	13·849	13·100	11·793
43	16·271	15·304	14·427	13·631	12·906	11·638
44	15·946	15·016	14·171	13·403	12·702	11·473
45	15·609	14·716	13·904	13·164	12·487	11·299
46	15·260	14·405	13·625	12·913	12·261	11·113
47	14·925	14·105	13·357	12·672	12·044	10·935
48	14·578	13·794	13·076	12·419	11·815	10·746
49	14·244	13·494	12·807	12·176	11·595	10·564
50	13·899	13·1[illegible]3	12·526	11·921	11·363	10·372
51	13·567	12·883	12·255	11·675	11·140	10·187
52	13·248	12·596	11·995	11·440	10·927	10·010
53	12·919	12·298	11·725	11·195	10·703	9·823
54	12·579	11·989	11·443	10·938	10·468	9·625
55	12·252	11·692	11·173	10·691	10·242	9·436
56	11·914	11·383	10·891	10·433	10·006	9·235
57	11·565	11·063	10·597	10·163	9·757	9·024
58	11·228	10·755	10·314	9·902	9·517	8·819
59	10·881	10·436	10·020	9·631	9·266	8·604
60	10·522	10·104	9·713	9·346	9·003	8·376
61	10·151	9·760	9·393	9·049	8·726	8·135
62	9·766	9·402	9·060	8·738	8·435	7·880
63	9·392	9·053	8·734	8·433	8·150	7·629
64	9·005	8·691	8·394	8·114	7·850	7·363
65	8·604	8·314	8·039	7·780	7·535	7·082
66	8·212	7·944	7·691	7·451	7·224	6·803
67	7·830	7·584	7·350	7·129	6·918	6·528
68	7·460	7·234	7·019	6·814	6·620	6·259
69	7·104	6·896	6·699	6·511	6·331	5·997
70	6·766	6·575	6·394	6·221	6·055	5·747
71	6·424	6·250	6·084	5·925	5·773	5·489
72	6·105	5·946	5·794	5·648	5·505	5·248
73	5·789	5·644	5·506	5·373	5·246	5·006
74	5·479	5·348	5·222	5·101	4·985	4·766
75	5·178	5·060	4·945	4·836	4·730	4·531

TABLE XV.

Showing the *Expectation of Life*, deduced from the observations made in *Sweden*.

Ages	Males	Females	Lives in general	Ages	Males	Females	Lives in general
1	42·45	44·00	42·95	31	29·69	31·54	30·57
2	43·83	46·05	44·92	32	29·04	30·91	29·94
3	44·96	47·31	46·11	33	28·39	30·28	29·30
4	45·57	48·04	46·78	34	27·74	29·66	28·67
5	45·62	48·00	46·79	35	27·09	29·03	28·03
6	45·50	47·87	46·66	36	26·43	28·26	27·31
7	45·26	47·64	46·43	37	25·76	27·50	26·68
8	44·91	47·28	46·07	38	25·09	26·74	26·01
9	44·46	46·80	45·61	39	24·42	25·97	25·33
10	43·94	46·25	45·07	40	23·75	25·21	24·66
11	43·26	45·55	44·38	41	23·15	24·68	24·05
12	42·58	44·85	43·70	42	22·54	24·75	23·44
13	41·91	44·15	43·01	43	21·93	23·62	22·83
14	41·24	43·46	42·33	44	21·32	23·10	22·22
15	40·56	42·76	41·64	45	20·71	22·57	21·61
16	39·83	42·04	40·92	46	20·12	21·91	20·98
17	39·11	41·31	40·19	47	19·52	21·24	20·35
18	38·39	40·59	39·47	48	18·92	20·58	19·72
19	37·67	39·87	38·74	49	18·32	19·92	19·09
20	36·95	39·15	38·02	50	17·72	19·26	18·46
21	36·28	38·43	37·33	51	17·17	18·64	17·87
22	35·62	37·72	36·64	52	16·63	18·01	17·29
23	34·96	37·01	35·96	53	16·08	17·39	16·70
24	34·30	36·29	35·27	54	15·53	16·77	16·12
25	33·63	35·58	34·58	55	14·98	16·15	15·53
26	32·98	34·90	33·91	56	14·43	15·53	14·95
27	32·32	34·21	33·23	57	13·87	14·92	14·37
28	31·66	33·53	32·56	58	13·33	14·31	13·79
29	31·00	32·85	31·88	59	12·79	13·69	13·21
30	30·34	32·17	31·21	60	12·24	13·08	12·63

TABLE XV Continued.

Ages	Males	Females	Lives in general	Ages	Males	Females	Lives in general
61	11·72	12·56	12·12	81	3·96	4·13	4·01
62	11·21	12·04	11·62	82	3·69	3·84	3·80
63	10·73	11·52	11·11	83	3·45	3·59	3·57
64	10·26	11·01	10·61	84	3·30	3·42	3·39
65	9·78	10·49	10·10	85	3·16	3·40	3·23
66	9·30	9·97	9·62	86	3·04	3·34	3·09
67	8·84	9·46	9·15	87	2·88	3·22	2·92
68	8·40	8·94	8·67	88	2·64	3·05	2·71
69	7·99	8·42	8·20	89	2·34	2·82	2·43
70	7·60	7·91	7·72	90	2·02	2·55	2·05
71	7·22	7·53	7·32	91	1·73	2·27	1·71
72	6·87	7·16	6·89	92	1·38	1·95	1·40
73	6·53	6·78	6·53	93	1·00	1·64	1·23
74	6·22	6·40	6·23	94	·75	1·35	1·10
75	5·89	6·03	5·91	95	·00	1·07	1·00
76	5·56	5·73	5·59	96	·00	·83	·00
77	5·25	5·43	5·28	97	·00	·00	·00
78	4·92	5·11	4·96				
79	4·59	4·79	4·61				
80	4·27	4·47	4·28				

TABLE XVI.

Showing the value of an annuity on a Single Life, at every age (Male or Female), deduced from the observations made in *Sweden*.

Ages	Males		Females	
	4 per Cent	5 per Cent	4 per Cent	5 per Cent
1	16·503	14·051	16·820	14·271
2	17·355	14·778	17·719	15·034
3	17·935	15·279	18·344	15·571
4	18·328	15·624	18·780	15·951
5	18·503	15·786	18·927	16·088
6	18·622	15·901	19·045	16·203
7	18·693	15·977	19·131	16·291
8	18·725	16·021	19·162	16·335
9	18·715	16·030	19·151	16·343
10	18·674	16·014	19·109	16·325
11	18·600	15·970	19·041	16·286
12	18·491	15·896	18·952	16·229
13	18·378	15·8[illegible]9	18·840	16·153
14	18·246	15·724	18·707	16·059
15	18·105	15·624	18·568	15·960
16	17·958	15·517	18·424	15·856
17	17·803	15·404	18·290	15·761
18	17·643	15·285	18·151	15·662
19	17·492	15·175	18·013	15·563
20	17·335	15·059	17·872	15·462
21	17·192	14·955	17·725	15·356
22	17·042	14·846	17·573	15·245
23	16·887	14·732	17·414	15·129
24	16·742	14·627	17·252	15·009
25	16·592	14·517	17·087	14·886
26	16·436	14·402	16·915	14·757
27	16·274	14·282	16·751	14·636
28	16·105	14·156	16·588	14·515
29	15·930	14·024	16·427	14·396
30	15·751	13·889	16·261	14·272

TABLE XVI Continued.

Ages	Males		Females	
	4 per Cent	5 per Cent	4 per Cent	5 per Cent
31	15·575	13·756	16·104	14·156
32	15·395	13·619	15·941	14·035
33	15·208	13·477	15·787	13·923
34	15·014	13·327	15·629	13·806
35	14·812	13·170	15·465	13·684
36	14·601	13·006	15·278	13·542
37	14·382	12·833	15·070	13·382
38	14·154	12·652	14·854	13·213
39	13·916	12·462	14·629	13·036
40	13·668	12·261	14·401	12·856
41	13·426	12·065	14·185	12·687
42	13·196	11·880	13·994	12·538
43	12·984	11·710	13·798	12·387
44	12·763	11·532	13·596	12·229
45	12·535	11·347	13·383	12·061
46	12·297	11·153	13·151	11·876
47	12·051	10·951	12·894	11·668
48	11·795	10·738	12·620	11·443
49	11·528	10·516	12·333	11·205
50	11·267	10·298	12·049	10·970
51	11·030	10·100	11·769	10·737
52	10·785	9·895	11·492	10·507
53	10·531	9·682	11·220	10·280
54	10·269	9·460	10·937	10·042
55	9·998	9·229	10·642	9·792
56	9·717	8·988	10·334	9·529
57	9·425	8·736	10·012	9·253
58	9·140	8·489	9·692	8·976
59	8·845	8·232	9·358	8·684
60	8·540	7·963	9·039	8·406
61	8·241	7·700	8·739	8·144
62	7·950	7·442	8·453	7·895
63	7·669	7·193	8·166	7·643
64	7·382	6·938	7·870	7·382
65	7·090	6·676	7·566	7·111

TABLE XVI Continued.

Ages	Males		Females	
	4 per Cent	5 per Cent	4 per Cent	5 per Cent
66	6·792	6·408	7·252	6·831
67	6·489	6·134	6·930	6·541
68	6·201	5·872	6·596	6·239
69	5·933	5·628	6·253	5·926
70	5·670	5·389	5·897	5·599
71	5·418	5·158	5·564	5·293
72	5·180	4·940	5·261	5·013
73	4·940	4·719	4·998	4·770
74	4·724	4·521	4·792	4·581
75	4·487	4·302	4·582	4·388
76	4·253	4·084	4·367	4·189
77	4·024	3·871	4·145	3·983
78	3·768	3·631	3·913	3·767
79	3·512	3·390	3·668	3·536
80	3·260	3·152	3·402	3·285
81	3·017	2·921	3·145	3·041
82	2·792	2·706	2·905	2·812
83	2·600	2·528	2·699	2·615
84	2·473	2·403	2·559	2·480
85	2·371	2·306	2·552	2·476
86	2·281	2·222	2·518	2·446
87	2·154	2·103	2·431	2·365
88	1·955	1·912	2·294	2·236
89	1·698	1·664	2·108	2·059
90	1·417	1·392	1·873	1·833
91	1·154	1·136	1·628	1·596
92	·835	·824	1·349	1·325
93	·477	·471	1·071	1·054
94	·240	·238	·799	·788
95	·000	·000	·544	·537
96	·000	·000	·320	·317

TABLE XVII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 0 years.

Common Age	4 per Cent	Common Age	4 per Cent	Common Age	4 per Cent	Common Age	4 per Cent
1	12·252	26	13·671	51	8·469	76	2·490
2	13·588	27	13·495	52	8·230	77	2·340
3	14·558	28	13·323	53	7·994	78	2·170
4	15·267	29	13·148	54	7·748	79	1·967
5	15·577	30	12·965	55	7·495	80	1·758
6	15·820	31	12·795	56	7·229	81	1·600
7	16·003	32	12·624	57	6·954	82	1·472
8	16·109	33	12·456	58	6·678	83	1·364
9	16·152	34	12·286	59	6·388	84	1·276
10	16·141	35	12·109	60	6·104	85	1·212
11	16·087	36	11·904	61	5·844	86	1·172
12	15·982	37	11·683	62	5·600	87	1·127
13	15·855	38	11·452	63	5·367	88	1·071
14	15·701	39	11·209	64	5·128	89	·949
15	15·535	40	10·964	65	4·881	90	·718
16	15·361	41	10·732	66	4·626	91	·516
17	15·196	42	10·531	67	4·362	92	·326
18	15·023	43	10·346	68	4·103	93	·236
19	14·854	44	10·154	69	3·851	94	·190
20	14·682	45	9·954	70	3·593	95	·024
21	14·525	46	9·736	71	3·345	96	·000
22	14·360	47	9·497	72	3·128		
23	14·194	48	9·236	73	2·935		
24	14·020	49	8·966	74	2·797		
25	13·849	50	8·707	75	2·648		

TABLE XVIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 6 years.

Ages	4 per Cent	Ages	4 per Cent	Ages	4 per Cent
1– 7	13·989	31–37	12·192	61–67	4·984
2– 8	14·780	32–38	11·988	62–68	4·729
3– 9	15·323	33–39	11·779	63–69	4·482
4–10	15·685	34–40	11·568	64–70	4·231
5–11	15·817	35–41	11·361	65–71	3·982
6–12	15·887	36–42	11·156	66–72	3·750
7–13	15·914	37–43	10·953	67–73	3·527
8–14	15·888	38–44	10·741	68–74	3·340
9–15	15·824	39–45	10·519	69–75	3·147
10–16	15·729	40–46	10·286	70–76	2·946
11–17	15·617	41–47	10·049	71–77	2·752
12–18	15·477	42–48	9·813	72–78	2·558
13–19	15·327	43–49	9·581	73–79	2·355
14–20	15·164	44–50	9·351	74–80	2·172
15–21	15·001	45–51	9·129	75–81	2·017
16–22	14·832	46–52	8·897	76–82	1·877
17–23	14·665	47–53	8·658	77–83	1·756
18–24	14·491	48–54	8·402	78–84	1·639
19–25	14·320	49–55	8·139	79–85	1·524
20–26	14·144	50–56	7·874	80–86	1·416
21–27	13·976	51–57	7·613	81–87	1·320
22–28	13·807	52–58	7·351	82–88	1·225
23–29	13·635	53–59	7·083	83–89	1·094
24–30	13·455	54–60	6·814	84–90	·902
25–31	13·284	55–61	6·555	85–91	·725
26–32	13·108	56–62	6·299	86–92	·556
27–33	12·935	57–63	6·045	87–93	·459
28–34	12·763	58–64	5·788	88–94	·396
29–35	12·586	59–65	5·519	89–95	·364
30–36	12·390	60–66	5·249	90–96	·000

TABLE XIX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 12 years.

Ages	4 per Cent	Ages	4 per Cent	Ages	4 per Cent
1–13	13·894	31–43	11·359	61–73	3·927
2–14	14·557	32–44	11·170	62–74	3·747
3–15	14·988	33–45	10·978	63–75	3·563
4–16	15·259	34–46	10·775	64–76	3·370
5–17	15·326	35–47	10·557	65–77	3·180
6–18	15·354	36–48	10·314	66–78	2·974
7–19	15·351	37–49	10·059	67–79	2·743
8–20	15·310	38–50	9·805	68–80	2·514
9–21	15·244	39–51	9·558	69–81	2·324
10–22	15·149	40–52	9·308	70–82	2·155
11–23	15·033	41–53	9·066	71–83	2·004
12–24	14·889	42–54	8·830	72–84	1·875
13–25	14·736	43–55	8·597	73–85	1·768
14–26	14·566	44–56	8·354	74–86	1·692
15–27	14·392	45–57	8·101	75–87	1·605
16–28	14·216	46–58	7·841	76–88	1·497
17–29	14·042	47–59	7·563	77–89	1·339
18–30	13·860	48–60	7·281	78–90	1·097
19–31	13·687	49–61	7·008	79–91	·863
20–32	13·512	50–62	6·749	80–92	·638
21–33	13·345	51–63	6·505	81–93	·511
22–34	13·173	52–64	6·256	82–94	·427
23–35	12·997	53–65	6·004	83–95	·379
24–36	12·801	54–66	5·743	84–96	·000
25–37	12·599	55–67	5·474		
26–38	12·387	56–68	5·204		
27–39	12·170	57–69	4·936		
28–40	11·953	58–70	4·664		
29–41	11·742	59–71	4·395		
30–42	11·543	60–72	4·149		

TABLE XX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 18 years.

Ages	4 per Cent	Ages	4 per Cent	Ages	4 per Cent
1–19	13·389	31–49	10·365	61–79	2·974
2–20	14·008	32–50	10·128	62–80	2·744
3–21	14·417	33–51	9·905	63–81	2·537
4–22	14·671	34–52	9·679	64–82	2·396
5–23	14·725	35–53	9·452	65–83	2·252
6–24	14·740	36–54	9·207	66–84	2·123
7–25	14·727	37–55	8·951	67–85	2·010
8–26	14·673	38–56	8·683	68–86	1·910
9–27	14·590	39–57	8·404	69–87	1·798
10–28	14·484	40–58	8·124	70–88	1·661
11–29	14·357	41–59	7·839	71–89	1·464
12–30	14·202	42–60	7·569	72–90	1·189
13–31	14·045	43–61	7·318	73–91	·937
14–32	13·874	44–62	7·075	74–92	·708
15–33	13·700	45–63	6·836	75–93	·575
16–34	13·520	46–64	6·586	76–94	·481
17–35	13·340	47–65	6·323	77–95	·421
18–36	13·141	48–66	6·048	78–96	·000
19–37	12·934	49–67	5·764		
20–38	12·720	50–68	5·487		
21–39	12·505	51–69	5·221		
22–40	12·286	52–70	4·953		
23–41	12·073	53–71	4·694		
24–42	11·873	54–72	4·455		
25–43	11·683	55–73	4·231		
26–44	11·485	56–74	4·043		
27–45	11·284	57–75	3·844		
28–46	11·072	58–76	3·637		
29–47	10·847	59–77	3·430		
30–48	10·606	60–78	3·210		

TABLE XXI.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 24 years.

Ages	4 per Cent	Ages	4 per Cent	Ages	4 per Cent
1–25	12·832	26–50	10·364	51–75	4·008
2–26	13·409	27–51	10·130	52–76	3·808
3–27	13·778	28–52	9·894	53–77	3·605
4–28	14·003	29–53	9·659	54–78	3·389
5–29	14·037	30–54	9·413	55–79	3·150
6–30	14·033	31–55	9·167	56–80	2·909
7–31	14·006	32–56	8·912	57–81	2·710
8–32	13·944	33–57	8·651	58–82	2·539
9–33	13·855	34–58	8·389	59–83	2·385
10–34	13·741	35–59	8·114	60–84	2·248
11–35	13·604	36–60	7·833	61–85	2·135
12–36	13·428	37–61	7·561	62–86	2·037
13–37	13·234	38–62	7·296	63–87	1·926
14–38	13·023	39–63	7·033	64–88	1·790
15–39	12·798	40–64	6·763	65–89	1·585
16–40	12·570	41–65	6·492	66–90	1·290
17–41	12·351	42–66	6·225	67–91	1·017
18–42	12·146	43–67	5·957	68–92	·764
19–43	11·951	44–68	5·689	69–93	·617
20–44	11·751	45–69	5·426	70–94	·514
21–45	11·550	46–70	5·153	71–95	·441
22–46	11·335	47–71	4·884	72–96	·000
23–47	11·107	48–72	4·633		
24–48	10·862	49–73	4·398		
25–49	10·612	50–74	4·205		

TABLE XXII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 30 years.

Ages	4 per Cent	Ages	4 per Cent	Ages	4 per Cent
1–31	12·196	26–56	9·080	51–81	2·792
2–32	12·730	27–57	8·807	52–82	2·623
3–33	13·066	28–58	8·534	53–83	2·475
4–34	13·264	29–59	8·250	54–84	2·344
5–35	13·277	30–60	7·967	55–85	2·232
6–36	13·242	31–61	7·702	56–86	2·130
7–37	13·170	32–62	7·446	57–87	2·010
8–38	13·059	33–63	7·196	58–88	1·864
9–39	12·913	34–64	6·942	59–89	1·644
10–40	12·743	35–65	6·679	60–90	1·333
11–41	12·563	36–66	6·402	61–91	1·050
12–42	12·379	37–67	6·115	62–92	·789
13–43	12·196	38–68	5·828	63–93	·639
14–44	11·997	39–69	5·543	64–94	·533
15–45	11·787	40–70	5·254	65–95	·456
16–46	11·562	41–71	4·977	66–96	·000
17–47	11·328	42–72	4·730		
18–48	11·076	43–73	4·507		
19–49	10·819	44–74	4·322		
20–50	10·567	45–75	4·128		
21–51	10·332	46–76	3·921		
22–52	10·092	47–77	3·715		
23–53	9·852	48–78	3·489		
24–54	9·602	49–79	3·238		
25–55	9·347	50–80	2·990		

TABLE XXIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 36 years.

Ages	4 per Cent	Ages	4 per Cent
1–37	11·465	31–67	6·197
2–38	11·913	32–68	5·917
3–39	12·164	33–69	5·642
4–40	12·284	34–70	5·364
5–41	12·242	35–71	5·093
6–42	12·185	36–72	4·840
7–43	12·112	37–73	4·603
8–44	12·004	38–74	4·405
9–45	11·865	39–75	4·195
10–46	11·694	40–76	3·975
11–47	11·493	41–77	3·762
12–48	11·259	42–78	3·539
13–49	11·011	43–79	3·295
14–50	10·759	44–80	3·052
15–51	10·514	45–81	2·854
16–52	10·264	46–82	2·684
17–53	10·018	47–83	2·533
18–54	9·761	48–84	2·396
19–55	9·500	49–85	2·277
20–56	9·228	50–86	2·171
21–57	8·953	51–87	2·050
22–58	8·675	52–88	1·901
23–59	8·385	53–89	1·681
24–60	8·097	54–90	1·366
25–61	7·823	55–91	1·078
26–62	7·557	56–92	·810
27–63	7·297	57–93	·655
28–64	7·032	58–94	·546
29–65	6·761	59–95	·464
30–66	6·481	60–96	·000

TABLE XXIV.

Showing the value of an annuity on Two Joint Lives, deduced from the observations (among Males and Females collectively) made in *Sweden*.

Difference of age 42 years.

Ages	4 per Cent	Ages	4 per Cent
1–43	10·546	31–73	4·646
2–44	10·946	32–74	4·453
3–45	11·168	33–75	4·251
4–46	11·260	34–76	4·040
5–47	11·183	35–77	3·833
6–48	11·064	36–78	3·605
7–49	10·915	37–79	3·352
8–50	10·743	38–80	3·098
9–51	10·560	39–81	2·889
10–52	10·357	40–82	2·710
11–53	10·140	41–83	2·553
12–54	9·898	42–84	2·418
13–55	9·644	43–85	2·305
14–56	9·371	44–86	2·203
15–57	9·087	45–87	2·083
16–58	8·799	46–88	1·933
17–59	8·503	47–89	1·708
18–60	8·208	48–90	1·385
19–61	7·928	49–91	1·090
20–62	7·658	50–92	·818
21–63	7·396	51–93	·662
22–64	7·127	52–94	·551
23–65	6·851	53–95	·468
24–66	6·566	54–96	·000
25–67	6·275		
26–68	5·986		
27–69	5·702		
28–70	5·415		
29–71	5·136		
30–72	4·881		

TABLE XXV.

Showing the number of persons *living* and *dying* at every age, according to the observations made at *Northampton*.

Age	Living	Dying	Age	Living	Dying	Ages	Living	Dying
0	11650	1340						
1	8650	1367	36	3935	75	71	1152	80
2	7283	502	37	3860	75	72	1072	80
3	6781	335	38	3785	75	73	992	80
4	6446	197	39	3710	75	74	912	80
5	6249	184	40	3635	76	75	832	80
6	6065	140	41	3559	77	76	752	77
7	5925	110	42	3482	78	77	675	73
8	5815	80	43	3404	78	78	602	68
9	5735	60	44	3326	78	79	534	65
10	5675	52	45	3248	78	80	469	63
11	5623	50	46	3170	78	81	406	60
12	5573	50	47	3092	78	82	346	57
13	5523	50	48	3014	78	83	289	55
14	5473	50	49	2936	79	84	234	48
15	5423	50	50	2857	81	85	186	41
16	5373	53	51	2776	82	86	145	34
17	5320	58	52	2694	82	87	111	28
18	5262	63	53	2612	82	88	83	21
19	5199	67	54	2530	82	89	62	16
20	5132	72	55	2448	82	90	46	12
21	5060	75	56	2366	82	91	34	10
22	4985	75	57	2284	82	92	24	8
23	4910	75	58	2202	82	93	16	7
24	4835	75	59	2120	82	94	9	5
25	4760	75	60	2038	82	95	4	3
26	4685	75	61	1956	82	96	1	1
27	4610	75	62	1874	81	97	0	0
28	4535	75	63	1793	81			
29	4460	75	64	1712	80			
30	4385	75	65	1632	80			
31	4310	75	66	1552	80			
32	4235	75	67	1472	80			
33	4160	75	68	1392	80			
34	4085	75	69	1312	80			
35	4010	75	70	1232	80			

TABLE XXVI.

Showing the *Expectations of Life* at every age, deduced from the observations made at *Northampton.*

Ages	Expectat.	Ages	Expectat.	Ages	Expectat.	Ages	Expectat.
1	32·74	26	30·33	51	17·50	76	6·18
2	37·79	27	29·82	52	17·02	77	5·83
3	39·55	28	29·30	53	16·54	78	5·48
4	40·58	29	28·79	54	16·06	79	5·11
5	40·84	30	28·27	55	15·58	80	4·75
6	41·07	31	27·76	56	15·10	81	4·41
7	41·03	32	27·24	57	14·63	82	4·09
8	40·79	33	26·72	58	14·15	83	3·80
9	40·36	34	26·20	59	13·68	84	3·58
10	39·78	35	25·68	60	13·21	85	3·37
11	39·14	36	25·16	61	12·75	86	3·19
12	38·49	37	24·64	62	12·28	87	3·01
13	37·83	38	24·12	63	11·81	88	2·86
14	37·17	39	23·60	64	11·35	89	2·66
15	36·51	40	23·08	65	10·88	90	2·41
16	35·85	41	22·56	66	10·42	91	2·09
17	35·20	42	22·04	67	9·96	92	1·75
18	34·58	43	21·54	68	9·50	93	1·37
19	33·99	44	21·03	69	9·05	94	1·05
20	33·43	45	20·52	70	8·60	95	·75
21	32·90	46	20·02	71	8·17	96	·50
22	32·39	47	19·51	72	7·74	97	·00
23	31·88	48	19·00	73	7·33		
24	31·36	49	18·49	74	6·92		
25	30·85	50	17·99	75	6·54		

TABLE XXVII.

Showing the value of an annuity on a Single Life, at every age, deduced from the observations made at *Northampton.*

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent	7 per Cent	8 per Cent
1	16·021	13·465	11·563	10·107	8·963	8·046
2	18·599	15 633	13·420	11·724	10·391	9·321
3	19·575	16·462	14·135	12·348	10·941	9·812
4	20·210	17·010	14·613	12·769	11·315	10·147
5	20·473	17·248	14·827	12·962	11·489	10·304
6	20·727	17·482	15·041	13·156	11·666	10·466
7	20·853	17·611	15·166	13·275	11·777	10·570
8	20·885	17·662	15·226	13·337	11 840	10·631
9	20·812	17·625	15·210	13·335	11·846	10·641
10	20·663	17·523	15·139	13·285	11·809	10·614
11	20·480	17·393	15·043	13·212	11·752	10·569
12	20·283	17·251	14·937	13·130	11·687	10·517
13	20·081	17·103	14·826	13·044	11·618	10·461
14	19·872	16·950	14·710	12·953	11·545	10·401
15	19·657	16·791	14·588	12·857	11·467	10·337
16	19·435	16 625	14·460	12·755	11·384	10·268
17	19·218	16 462	14·334	12·655	11·302	10·200
18	19·013	16 309	14·217	12·562	11·226	10·137
19	18·820	16·167	14 108	12·477	11·157	10·081
20	18·638	16 033	14·007	12·398	11·094	10·030
21	18·470	15·912	13·917	12·329	11·042	9·986
22	18·311	15·797	13·833	12·265	10·993	9·947
23	18·148	15·680	13·746	12·200	10·942	9·907
24	17·983	15·560	13 658	12·132	10·890	9·865
25	17·814	15·438	13·567	12·063	10·836	9·823
26	17·642	15·312	13·473	11·992	10·780	9·778
27	17·467	15·184	13·377	11 917	10·723	9·732
28	17·289	15·053	13·278	11·841	10·663	9·685
29	17·107	14·918	13·177	11·763	10·602	9·635
30	16·922	14·781	13·072	11·682	10·539	9 584

TABLE XXVII Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent	7 per Cent	8 per Cent
31	16·732	14·639	12·965	11·598	10·473	9·531
32	16·540	14·495	12·854	11·512	10·404	9·476
33	16·343	14·347	12·740	11·423	10·333	9·418
34	16·142	14·195	12·623	11·331	10·260	9·359
35	15·938	14·039	12·502	11·236	10·183	9·296
36	15·729	13·880	12·377	11·137	10·104	9·231
37	15·515	13·716	12·249	11·035	10·021	9·164
38	15·298	13·548	12·116	10·929	9·935	9·093
39	15·075	13·375	11·979	10·819	9·845	9·019
40	14·848	13·197	11·837	10·705	9·752	8·941
41	14·620	13·018	11·695	10·589	9·657	8·863
42	14·391	12·838	11·551	10·473	9·562	8·783
43	14·162	12·657	11·407	10·356	9·466	8·703
44	13·929	12·472	11·258	10·235	9·366	8·620
45	13·692	12·283	11·105	10·110	9·262	8·533
46	13·450	12·089	10·947	9·980	9·154	8·443
47	13·203	11·890	10·784	9·846	9·042	8·348
48	12·951	11·685	10·616	9·707	8·925	8·249
49	12·693	11·475	10·443	9·563	8·804	8·146
50	12·436	11·264	10·269	9·417	8·681	8·041
51	12·183	11·057	10·097	9·273	8·559	7·937
52	11·930	10·849	9·925	9·129	8·437	7·833
53	11·674	10·637	9·748	8·980	8·311	7·725
54	11·414	10·421	9·567	8·827	8·181	7·614
55	11·150	10·201	9·382	8·670	8·047	7·499
56	10·882	9·977	9·193	8·509	7·909	7·379
57	10·611	9·749	8·999	8·343	7·766	7·256
58	10·337	9·516	8·801	8·173	7·619	7·128
59	10·058	9·280	8·599	7·999	7·468	6·996
60	9·777	9·039	8·392	7·820	7·312	6·860
61	9·493	8·795	8·181	7·637	7·152	6·719
62	9·205	8·547	7·966	7·449	6·988	6·574
63	8·910	8·291	7·742	7·253	6·815	6·421
64	8·611	8·030	7·514	7·052	6·637	6·262
65	8·304	7·761	7·276	6·841	6·449	6·095

TABLE XXVII Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent	7 per Cent	8 per Cent
66	7·994	7·488	7·034	6·625	6·256	5·922
67	7·682	7·211	6·787	6·405	6·058	5·743
68	7·367	6·930	6·536	6·179	5·855	5·559
69	7·051	6·647	6·281	5·949	5·646	5·370
70	6·734	6·361	6·023	5·716	5·434	5·176
71	6·418	6·075	5·764	5·479	5·218	4·978
72	6·103	5·790	5·504	5·241	5·000	4·778
73	5·794	5·507	5·245	5·004	4·781	4·576
74	5·491	5·230	4·990	4·769	4·565	4·375
75	5·199	4·962	4·744	4·542	4·354	4·180
76	4·925	4·710	4·511	4·326	4·154	3·994
77	4·652	4·457	4·277	4·109	3·952	3·806
78	4·372	4·197	4·035	3·884	3·742	3·609
79	4·077	3·921	3·776	3·641	3·514	3·394
80	3·761	3·643	3·515	3·394	3·281	3·174
81	3·499	3·377	3·263	3·156	3·055	2·960
82	3·229	3·122	3·020	2·926	2·836	2·751
83	2·962	2·887	2·797	2·713	2·632	2·557
84	2·793	2·708	2·627	2·551	2·479	2·410
85	2·620	2·543	2·471	2·402	2·337	2·275
86	2·462	2·393	2·328	2·266	2·207	2·151
87	2·312	2·251	2·193	2·138	2·085	2·035
88	2·185	2·131	2·080	2·031	1·984	1·939
89	2·013	1·967	1·924	1·882	1·842	1·803
90	1·794	1·758	1·723	1·689	1·656	1·625
91	1·501	1·474	1·447	1·422	1·398	1·374
92	1·190	1·171	1·153	1·136	1·118	1·102
93	·839	·827	·816	·806	·795	·785
94	·536	·530	·524	·518	·512	·507
95	·242	·240	·238	·236	·234	·232
96	·000	·000	·000	·000	·000	·000

TABLE XXVIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 0 years.

Common Age	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1	9·491	8·252	7·287	6·515
2	12·789	11·107	9·793	8·741
3	14·196	12·325	10·862	9·689
4	15·181	13·185	11·621	10·365
5	15·638	13·591	11·984	10·691
6	16·099	14·005	12·358	11·031
7	16·375	14·224	12·596	11·251
8	16·510	14·399	12·731	11·382
9	16·483	14·396	12·744	11·404
10	16·339	14·277	12·665	11·345
11	16·142	14·133	12·546	11·249
12	15·926	13·966	12·411	11·139
13	15·702	13·789	12·268	11·023
14	15·470	13·604	12·118	10·899
15	15·229	13·411	11·960	10·767
16	14·979	13·212	11·793	10·626
17	14·737	13·019	11·630	10·489
18	14·516	12·841	11·483	10·365
19	14·316	12·679	11·351	10·255
20	14·133	12·535	11·232	10·156
21	13·974	12·409	11·131	10·074
22	13·830	12·293	11·042	10·002
23	13·683	12·179	10·951	9·928
24	13·534	12·062	10·858	9·853
25	13·383	11·944	10·764	9·776
26	13·230	11·822	10·667	9·697
27	13·074	11 699	10·567	9·616
28	12·915	11·573	10·466	9·533
29	12·754	11·445	10·362	9·448
30	12·589	11·313	10·255	9·360

TABLE XXVIII Continued.

Common Age	3 per Cent.	4 per Cent	5 per Cent	6 per Cent
31	12·422	11·179	10·146	9·270
32	12·252	11·042	10·034	9·178
33	12·079	10·902	9·919	9·082
34	11·902	10·759	9·801	8·984
35	11·722	10·612	9·680	8·883
36	11·539	10·462	9·555	8·778
37	11·351	10·307	9·427	8·670
38	11·160	10·149	9·294	8·558
39	10·964	9·986	9·158	8·442
40	10·764	9·820	9·016	8·322
41	10·565	9·654	8·876	8·202
42	10·369	9·491	8·737	8·083
43	10·175	9·326	8·599	7·965
44	9·978	9·160	8·457	7·843
45	9·776	8·990	8·312	7·718
46	9·571	8·815	8·162	7·589
47	9·362	8·637	8·008	7·455
48	9·149	8·453	7·849	7·316
49	8·931	8·266	7·686	7·173
50	8·714	8·081	7·522	7·030
51	8·507	7·900	7·366	6·893
52	8·304	7·724	7·213	6·758
53	8·099	7·544	7·056	6·620
54	7·891	7·362	6·897	6·480
55	7·681	7·179	6·735	6·336
56	7·470	6·993	6·571	6·190
57	7·256	6·805	6·404	6·041
58	7·041	6·614	6·234	5·890
59	6·824	6·421	6·062	5·735
60	6·606	6·226	5·888	5·579
61	6·387	6·030	5·712	5·420
62	6·166	5·831	5·533	5·259
63	5·938	5·626	5·347	5·089
64	5·709	5·417	5·158	4·917
65	5·471	5·201	4·960	4·736

TABLE XXVIII Continued.

Common Age	3 per Cent	4 per Cent	5 per Cent	6 per Cent
66	5·231	4·982	4·759	4·551
67	4·990	4·760	4·555	4·363
68	4·747	4·537	4·348	4·171
69	4·504	4·312	4·140	3·977
70	4·261	4·087	3·930	3·781
71	4·020	3·862	3·719	3·584
72	3·781	3·639	3·510	3·387
73	3·548	3·421	3·304	3·193
74	3·324	3·211	3·105	3·005
75	3·114	3·015	2·917	2·827
76	2·926	2·833	2·750	2·668
77	2·741	2·656	2·583	2·511
78	2·550	2·470	2·410	2·346
79	2·338	2·271	2·217	2·161
80	2·122	2·068	2·018	1·969
81	1·917	1·869	1·827	1·786
82	1·719	1·681	1·642	1·606
83	1·538	1·510	1·472	1·441
84	1·416	1·387	1·357	1·330
85	1·309	1·339	1·256	1·232
86	1·218	1·195	1·171	1·149
87	1·141	1·124	1·098	1·078
88	1·103	1·030	1·063	1·044
89	1·036	1·015	1·001	·984
90	·938	·922	·909	·895
91	·769	·756	·748	·737
92	·591	·583	·576	·569
93	·369	·365	·361	·357
94	·203	·201	·199	·197
95	·060	·060	·059	·058
96	·000	·000	·000	·000

TABLE XXIX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 5 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1– 6	12·347	10·741	9·479	8·467
2– 7	14·461	12·581	11·100	9·911
3– 8	15·300	13·319	11·755	10·498
4– 9	15·809	13·775	12·165	10·869
5–10	15·974	13·933	12·315	11·010
6–11	16·110	14·068	12·447	11·136
7–12	16·137	14·111	12·498	11·192
8–13	16·089	14·089	12·492	11·197
9–14	15·957	13·992	12·421	11·144
10–15	15·762	13·841	12·302	11·048
11–16	15·538	13·664	12·158	10·929
12–17	15·308	13·480	12·009	10·805
13–18	15·086	13·303	11·864	10·685
14–19	14·870	13·130	11·723	10·568
15–20	14·660	12·961	11·585	10·453
16–21	14·457	12·799	11·452	10·342
17–22	14·265	12·646	11·327	10·239
18–23	14·082	12·500	11·209	10·140
19–24	13·908	12·361	11·096	10·048
20–25	13·741	12·229	10·989	9·960
21–26	13·584	12·105	10·890	9·879
22–27	13·433	11·987	10·796	9·803
23–28	13·280	11·866	10·699	9·724
24–29	13·124	11·743	10·600	9·643
25–30	12·966	11·618	10·499	9·561
26–31	12·805	11·489	10·396	9·476
27–32	12·641	11·359	10·289	9·389
28–33	12·474	11·225	10·181	9·299
29–34	12·304	11·088	10·069	9·207
30–35	12·131	10·948	9·954	9·112

TABLE XXIX Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–36	11·955	10·805	9·837	9·014
32–37	11·775	10·659	9·716	8·913
33–38	11·592	10·508	9·591	8·808
34–39	11·404	10·354	9·463	8·701
35–40	11·213	10·196	9·331	8·589
36–41	11·021	10·037	9·198	8·476
37–42	10·828	9·877	9·062	8·362
38–43	10·635	9·716	8·927	8·246
39–44	10·437	9·550	8·787	8·127
40–45	10·236	9·381	8·643	8·003
41–46	10·033	9·210	8·497	7·878
42–47	9·829	9·037	8·350	7·751
43–48	9·624	8·862	8·200	7·621
44–49	9·414	8·683	8·046	7·488
45–50	9·204	8·503	7·891	7·353
46–51	8·997	8·326	7·737	7·219
47–52	8·790	8·147	7·582	7·084
48–53	8·579	7·965	7·424	6·945
49–54	8·366	7·780	7·262	6·802
50–55	8·152	7·593	7·098	6·658
51–56	7·941	7·409	6·936	6·515
52–57	7·730	7·225	6·774	6·371
53–58	7·518	7·039	6·609	6·225
54–59	7·304	6·850	6·442	6·076
55–60	7·088	6·659	6·272	5·924
56–61	6·870	6·465	6·100	5·770
57–62	6·651	6·270	5·925	5·613
58–63	6·427	6·070	5·744	5·450
59–64	6·201	5·867	5·561	5·284
60–65	5·970	5·658	5·372	5·112
61–66	5·737	5·447	5·180	4·938
62–67	5·503	5·285	4·986	4·760
63–68	5·265	5·017	4·786	4·576
64–69	5·025	4·798	4·585	4·390
65–70	4·783	4·573	4·378	4·199

TABLE XXIX Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
66–71	4·540	4·349	4·169	4·005
67–72	4·298	4·124	3·960	3·811
68–73	4·059	3·901	3·752	3·616
69–74	3·825	3·683	3·547	3·423
70–75	3·599	3·471	3·347	3·236
71–76	3·386	3·270	3·159	3·059
72–77	3·176	3·070	2·971	2·882
73–78	2·963	2·869	2·780	2·701
74–79	2·743	2·659	2·580	2·511
75–80	2·526	2·448	2·381	2·323
76–81	2·325	2·258	2·195	2·147
77–82	2·131	2·077	2·013	1·975
78–83	1·947	1·899	1·838	1·810
79–84	1·793	1·751	1·750	1·672
80–85	1·645	1·608	1·573	1·539
81–86	1·511	1·478	1·447	1·417
82–87	1·385	1·356	1·329	1·303
83–88	1·284	1·259	1·235	1·212
84–89	1·188	1·164	1·145	1·124
85–90	1·074	1·054	1·038	1·021
86–91	·921	·902	·892	·879
87–92	·756	·738	·734	·725
88–93	·562	·554	·547	·541
89–94	·377	·373	·369	·365
90–95	·179	·177	·175	·174
91–96	·000	·000	·000	·000

TABLE XXX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton*.

Difference of age 10 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–11	12·346	10·782	9·544	8·547
2–12	14·239	12·438	11·010	9·857
3–13	14·895	13·019	11·528	10·324
4–14	15·287	13·374	11·850	10·617
5–15	15·391	13·479	11·954	10·716
6–16	15·486	13·578	12·052	10·812
7–17	15·490	13·599	12·083	10·849
8–18	15·436	13·569	12·070	10·847
9–19	15·316	13·482	12·006	10·799
10–20	15·151	13·355	11·906	10·719
11–21	14·974	13·217	11·797	10·631
12–22	14·795	13·078	11·686	10·541
13–23	14·612	12·934	11·570	10·446
14–24	14·424	12·784	11·450	10·348
15–25	14·230	12·630	11·324	10·244
16–26	14·030	12·470	11·193	10·135
17–27	13·832	12·311	11·063	10·027
18–28	13·642	12·158	10·939	9·924
19–29	13·461	12·013	10·820	9·826
20–30	13·286	11·873	10·707	9·732
21–31	13·121	11·742	10·600	9·644
22–32	12·961	11·615	10·498	9·561
23–33	12·798	11·485	10·393	9·474
24–34	12·632	11·352	10·285	9·386
25–35	12·463	11·217	10·175	9·295
26–36	12·291	11·078	10·062	9·201
27–37	12·116	10·936	9·946	9·105
28–38	11·937	10·791	9·826	9·005
29–39	11·755	10·642	9·703	8·902
30–40	11·568	10·490	9·576	8·795

TABLE XXX Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–41	11·382	10·336	9·448	8·688
32–42	11·195	10·182	9·320	8·580
33–43	11·007	10·027	9·190	8·471
34–44	10·817	9·869	9·058	8·358
35–45	10·622	9·706	8·921	8·242
36–46	10·424	9·540	8·781	8·122
37–47	10·221	9·370	8·636	7·998
38–48	10·014	9·195	8·487	7·870
39–49	9·803	9·015	8·333	7·737
40–50	9·590	8·834	8·177	7·602
41–51	9·383	8·658	8·025	7·470
42–52	9·179	8·483	7·875	7·340
43–53	8·975	8·308	7·724	7·208
44–54	8·767	8·130	7·569	7·073
45–55	8·557	7·948	7·411	6·935
46–56	8·344	7·763	7·249	6·793
47–57	8·127	7·574	7·084	6·648
48–58	7·907	7·382	6·915	6·498
49–59	7·684	7·186	6·742	6·344
50–60	7·461	6·989	6·568	6·189
51–61	7·240	6·795	6·395	6·035
52–62	7·021	6·600	6·222	5·880
53–63	6·795	6·399	6·042	5·719
54–64	6·568	6·196	5·860	5·555
55–65	6·334	5·986	5·671	5·384
56–66	6·098	5·774	5·479	5·209
57–67	5·860	5·559	5·283	5·031
58–68	5·621	5·341	5·084	4·849
59–69	5·380	5·121	4·883	4·665
60–70	5·139	4·900	4·680	4·478
61–71	4·898	4·679	4·476	4·289
62–72	4·659	4·458	4·272	4·099
63–73	4·420	4·236	4·066	3·908
64–74	4·186	4·019	3·864	3·719
65–75	3·958	3·806	3·665	3·533

TABLE XXX Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
66–76	3·743	3.606	3·477	3·357
67–77	3·529	3·405	3·289	3·180
68–78	3·310	3·199	3·095	2·996
69–79	3·077	2·979	2·887	2·799
70–80	2·843	2·757	2·675	2·598
71–81	2·618	2·542	2·470	2·402
72–82	2·401	2·334	2·271	2·211
73–83	2·199	2·141	2·085	2·032
74–84	2·043	1·991	1·941	1·894
75–85	1·903	1·856	1·811	1·769
76–86	1·781	1·739	1·699	1·661
77–87	1·670	1·633	1·597	1·562
78–88	1·580	1·546	1·514	1·483
79–89	1·456	1·427	1·400	1·373
80–90	1·302	1·278	1·255	1·234
81–91	1·096	1·078	1·061	1·044
82–92	·877	·864	·852	·840
83–93	·622	·614	·606	·599
84–94	·408	·403	·398	·394
85–95	·189	·187	·185	·183
86–96	·000	·000	·000	·000

TABLE XXXI.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton*.

Difference of age 15 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–16	11·864	10·406	9·243	8·301
2–17	13·659	11·981	10·642	9·555
3–18	14·277	12·531	11·134	9·998
4–19	14·657	12·876	11·447	10·284
5–20	14·776	12·993	11·561	10·391
6–21	14·904	13·121	11·685	10·510
7–22	14·950	13·178	11·748	10·576
8–23	14·929	13·178	11·761	10·597
9–24	14·834	13·112	11·715	10·566
10–25	14·683	12·998	11·627	10·497
11–26	14·508	12·861	11·519	10·410
12–27	14·323	12·715	11·402	10·314
13–28	14·132	12·564	11·280	10·215
14–29	13·936	12·408	11·153	10·110
15–30	13·734	12·246	11·021	10·001
16–31	13·527	12·078	10·883	9·886
17–32	13·320	11·911	10·746	9·771
18–33	13·121	11·750	10·613	9·660
19–34	12·930	11·595	10·486	9·554
20–35	12·744	11·415	10·363	9·451
21–36	12·567	11·302	10·246	9·354
22–37	12·394	11·163	10·132	9·260
23–38	12·218	11·020	10·015	9·163
24–39	12·038	10·874	9·895	9·063
25–40	11·854	10·725	9·771	8·960
26–41	11·670	10·574	9·647	8·855
27–42	11·486	10·423	9·522	8·751
28–43	11·302	10·272	9·396	8·645
29–44	11·114	10·117	9·267	8·536
30–45	10·923	9·959	9·135	8·424

TABLE XXXI Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–46	10·728	9·797	8·998	8·309
32–47	10·530	9·631	8·858	8·189
33–48	10·327	9·461	8·714	8·066
34–49	10·120	9·286	8·565	7·938
35–50	9·912	9·110	8·415	7·809
36–51	9·707	8·937	8·267	7·681
37–52	9·503	8·763	8·119	7·553
38–53	9·296	8·586	7·966	7·421
39–54	9·085	8·406	7·810	7·286
40–55	8·870	8·221	7·651	7·146
41–56	8·655	8·035	7·489	7·005
42–57	8·439	7·848	7·326	6·862
43–58	8·222	7·660	7·162	6·718
44–59	8·003	7·469	6·994	6·570
45–60	7·781	7·274	6·822	6·418
46–61	7·556	7·076	6·648	6·263
47–62	7·328	6·875	6·469	6·104
48–63	7·093	6·667	6·283	5·937
49–64	6·854	6·454	6·093	5·767
50–65	6·611	6·236	5·897	5·590
51–66	6·369	6·019	5·701	5·412
52–67	6·127	5·801	5·504	5·233
53–68	5·884	5·580	5·303	5·050
54–69	5·638	5·357	5·100	4·864
55–70	5·391	5·132	4·893	4·674
56–71	5·145	4·905	4·685	4·482
57–72	4·899	4·679	4·477	4·289
58–73	4·656	4·455	4·269	4·096
59–74	4·418	4·234	4·064	3·906
60–75	4·189	4·021	3·866	3·721
61–76	3·974	3·821	3·679	3·546
62–77	3·760	3·621	3·492	3·371
63–78	3·538	3·414	3·297	3·188
64–79	3·303	3·192	3·088	2·990
65–80	3·063	2·965	2·873	2·786

TABLE XXXI Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
66–81	2·833	2·746	2·664	2·587
67–82	2·610	2·533	2·461	2·393
68–83	2·403	2·336	2·272	2·211
69–84	2·244	2·183	2·126	2·071
70–85	2·097	2·042	1·991	1·941
71–86	1·963	1·914	1·867	1·823
72–87	1·838	1·794	1·753	1·713
73–88	1·736	1·697	1·660	1·625
74–89	1·603	1·570	1·538	1·508
75–90	1·440	1·413	1·387	1·361
76–91	1·221	1·200	1·180	1·160
77–92	·985	·970	·955	·942
78–93	·706	·697	·688	·679
79–94	·458	·453	·448	·443
80–95	·210	·208	·206	·204
81–96	·000	·000	·000	·000

TABLE XXXII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton*.

Difference of age 20 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–21	11·413	10·053	8·961	8·070
2–22	13·172	11·605	10·344	9·313
3–23	13·794	12·161	10·843	9·764
4–24	14·178	12·511	11·163	10·057
5–25	14·301	12·633	11·281	10·170
6–26	14·420	12·754	11·400	10·285
7–27	14·451	12·798	11·452	10·341
8–28	14·417	12·786	11·455	10·354
9–29	14·310	12·710	11·401	10·315
10–30	14·150	12·586	11·304	10·239
11–31	13·965	12·441	11·188	10·144
12–32	13·770	12·286	11·062	10·042
13–33	13·570	12·125	10·932	9·934
14–34	13·363	11·959	10·796	9·822
15–35	13·151	11·787	10·655	9·703
16–36	12·932	11·609	10·507	9·579
17–37	12·714	11·430	10·358	9·454
18–38	12·502	11·257	10·214	9·333
19–39	12·297	11·089	10·074	9·215
20–40	12·096	10·924	9·937	9·100
21–41	11·906	10·768	9·809	8·992
22–42	11·723	10·619	9·685	8·889
23–43	11·540	10·470	9·562	8·785
24–44	11·354	10·317	9·435	8·670
25–45	11·164	10·160	9·304	8·569
26–46	10·970	10·000	9·170	8·455
27–47	10·773	9·836	9·032	8·338
28–48	10·572	9·667	8·890	8·217
29–49	10·366	9·495	8·744	8·092
30–50	10·160	9·321	8·596	7·966
31–51	9·957	9·151	8·451	7·841
32–52	9·756	8·980	8·306	7·716
33–53	9·550	8·806	8·157	7·588
34–54	9·342	8·629	8·005	7·457
35–55	9·131	8·448	7·849	7·322

TABLE XXXII Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
36–56	8·916	8·264	7·690	7·183
37–57	8·699	8·076	7·527	7·041
38–58	8·477	7·884	7·360	6·894
39–59	8·253	7·689	7·189	6·744
40–60	8·025	7·490	7·015	6·590
41–61	7·796	7·290	6·838	6·434
42–62	7·567	7·068	6·660	6·276
43–63	7·332	6·881	6·477	6·112
44–64	7·095	6·671	6·289	5·944
45–65	6·850	6·453	6·094	5·769
46–66	6·602	6·230	5·894	5·588
47–67	6·351	6·004	5·690	5·403
48–68	6·096	5·774	5·481	5·213
49–69	5·839	5·541	5·268	5·019
50–70	5·582	5·306	5·054	4·822
51–71	5·328	5·074	4·841	4·626
52–72	5·077	4·845	4·630	4·430
53–73	4·829	4·614	4·417	4·234
54–74	4·585	4·389	4·208	4·040
55–75	4·350	4·171	4·006	3·852
56–76	4·129	3·966	3·815	3·674
57–77	3·908	3·761	3·623	3·494
58–78	3·682	3·549	3·424	3·308
59–79	3·440	3·322	3·210	3·105
60–80	3·197	3·092	2·992	2·899
61–81	2·964	2·870	2·782	2·699
62–82	2·739	2·656	2·578	2·504
63–83	2·530	2·457	2·387	2·321
64–84	2·371	2·305	2·242	2·182
65–85	2·223	2·163	2·107	2·053
66–86	2·089	2·035	1·984	1·936
67–87	1·963	1·915	1·870	1·826
68–88	1·860	1·817	1·777	1·737
69–89	1·722	1·685	1·650	1·616
70–90	1·545	1·515	1·486	1·459
71–91	1·303	1·280	1·259	1·238
72–92	1·044	1·028	1·012	·997
73–93	·743	·733	·723	·714
74–94	·480	·474	·469	·464
75–95	·219	·217	·215	·213

TABLE XXXIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 25 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–26	11·037	9·770	8·742	7·897
2–27	12·722	11·264	10·080	9·104
3–28	13·307	11·790	10·555	9·537
4–29	13·661	12·116	10·855	9·813
5–30	13·762	12·220	10·959	9·918
6–31	13·859	12·322	11·062	10·015
7–32	13·871	12·350	11·100	10·060
8–33	13·820	12·323	11·090	10·061
9–34	13·698	12·234	11·024	10·012
10–35	13·525	12·098	10·916	9·925
11–36	13·328	11·941	10·788	9·820
12–37	13·120	11·773	10·651	9·707
13–38	12·906	11·600	10·509	9·588
14–39	12·686	11·420	10·360	9·464
15–40	12·459	11·234	10·205	9·333
16–41	12·229	11·044	10·046	9·198
17–42	12·002	10·856	9·889	9·065
18–43	11·785	10·677	9·739	8·938
19–44	11·574	10·502	9·592	8·814
20–45	11·367	10·330	9·448	8·692
21–46	11·167	10·165	9·310	8·574
22–47	10·969	10·001	9·173	8·458
23–48	10·768	9·833	9·031	8·338
24–49	10·562	9·661	8·886	8·214
25–50	10·356	9·488	8·739	8·089
26–51	10·154	9·318	8·595	7·966
27–52	9·952	9·148	8·451	7·842
28–53	9·748	8·975	8·304	7·716
29–54	9·540	8·799	8·153	7·586
30–55	9·329	8·619	7·999	7·453

TABLE XXXIII Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–56	9·115	8·436	7·841	7·316
32–57	8·897	8·250	7·680	7·175
33–58	8·677	8·060	7·515	7·031
34–59	8·454	7·866	7·346	6·884
35–60	8·227	7·669	7·174	6·732
36–61	7·997	7·469	6·998	6·577
37–62	7·765	7·265	6·819	6·418
38–63	7·525	7·053	6·631	6·252
39–64	7·281	6·838	6·440	6·081
40–65	7·030	6·614	6·240	5·901
41–66	6·776	6·388	6·037	5·718
42–67	6·522	6·159	5·831	5·532
43–68	6·266	5·929	5·622	5·343
44–69	6·008	5·696	5·411	5·150
45–70	5·749	5·460	5·195	4·953
46–71	5·488	5·222	4·978	4·753
47–72	5·228	4·983	4·758	4·551
48–73	4·970	4·746	4·539	4·348
49–74	4·716	4·511	4·322	4·146
50–75	4·472	4·285	4·112	3·951
51–76	4·245	4·074	3·916	3·768
52–77	4·019	3·864	3·720	3·586
53–78	3·787	3·648	3·518	3·396
54–79	3·540	3·416	3·299	3·189
55–80	3·291	3·180	3·076	2·978
56–81	3·051	2·953	2·861	2·774
57–82	2·820	2·733	2·651	2·574
58–83	2·608	2·530	2·457	2·388
59–84	2·446	2·376	2·310	2·247
60–85	2·297	2·234	2·174	2·118
61–86	2·162	2·105	2·051	2·000
62–87	2·036	1·985	1·937	1·891
63–88	1·932	1·886	1·843	1·802
64–89	1·790	1·751	1·714	1·678
65–90	1·606	1·575	1·544	1·515
66–91	1·354	1·330	1·307	1·285
67–92	1·083	1·067	1·050	1·035
68–93	·770	·760	·750	·740
69–94	·497	·491	·485	·480
70–95	·227	·224	·222	·220
71–96	·000	·000	·000	·000

TABLE XXXIV.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 30 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–31	10·605	9·438	8·483	7·691
2–32	12·203	10·865	9·767	8·855
3–33	12·743	11·355	10·213	9·263
4–34	13·061	11·651	10·488	9·518
5–35	13·136	11·732	10·572	9·602
6–36	13·207	11·812	10·656	9·687
7–37	13·195	11·819	10·676	9·715
8–38	13·122	11·772	10·648	9·701
9–39	12·981	11·665	10·565	9·637
10–40	12·791	11·513	10·442	9·537
11–41	12·580	11·342	10·302	9·420
12–42	12·363	11·165	10·156	9·298
13–43	12·144	10·985	10·007	9·173
14–44	11·918	10·799	9·852	9·042
15–45	11·687	10·607	9·690	8·905
16–46	11·448	10·408	9·522	8·762
17–47	11·210	10·208	9·353	8·617
18–48	10·975	10·011	9·186	8·473
19–49	10·746	9·818	9·021	8·332
20–50	10·523	9·630	8·861	8·195
21–51	10·313	9·454	8·712	8·067
22–52	10·111	9·284	8·568	7·944
23–53	9·905	9·111	8·421	7·818
24–54	9·696	8·934	8·270	7·688
25–55	9·484	8·754	8·116	7·555
26–56	9·269	8·570	7·958	7·419
27–57	9·051	8·383	7·797	7·279
28–58	8·830	8·193	7·632	7·135
29–59	8·605	7·999	7·464	6·988
30–60	8·378	7·802	7·292	6·837

TABLE XXXIV Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–61	8·147	7·601	7·116	6·682
32–62	7·914	7·397	6·937	6·524
33–63	7·673	7·186	6·750	6·359
34–64	7·429	6·971	6·559	6·189
35–65	7·177	6·747	6·360	6·010
36–66	6·922	6·520	6·156	5·827
37–67	6·663	6·288	5·948	5·639
38–68	6·401	6·052	5·735	5·446
39–69	6·137	5·813	5·518	5·249
40–70	5·871	5·571	5·298	5·047
41–71	5·605	5·329	5·076	4·844
42–72	5·341	5·087	4·854	4·640
43–73	5·081	4·848	4·634	4·436
44–74	4·826	4·613	4·417	4·235
45–75	4·580	4·386	4·206	4·040
46–76	4·348	4·171	4·006	3·853
47–77	4·115	3·954	3·805	3·666
48–78	3·875	3·731	3·596	3·469
49–79	3·619	3·490	3·369	3·256
50–80	3·362	3·247	3·140	3·039
51–81	3·117	3·015	2·920	2·829
52–82	2·882	2·792	2·707	2·627
53–83	2·665	2·585	2·510	2·438
54–84	2·501	2·428	2·360	2·295
55–85	2·349	2·284	2·222	2·164
56–86	2·211	2·153	2·097	2·044
57–87	2·082	2·030	1·980	1·932
58–88	1·975	1·928	1·883	1·841
59–89	1·828	1·788	1·750	1·713
60–90	1·641	1·608	1·577	1·547
61–91	1·382	1·358	1·334	1·311
62–92	1·105	1·088	1·071	1·055
63–93	·785	·774	·764	·754
64–94	·506	·500	·494	·489
65–95	·230	·228	·226	·224
66–96	·000	·000	·000	·000

TABLE XXXV.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 35 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–36	10·104	9·047	8·173	7·442
2–37	11·600	10·392	9·390	8·551
3–38	12·087	10·838	9·800	8·928
4–39	12·362	11·097	10·043	9·157
5–40	12·405	11·150	10·102	9·219
6–41	12·446	11·203	10·163	9·283
7–42	12·412	11·190	10·165	9·296
8–43	12·325	11·130	10·124	9·270
9–44	12·174	11·012	10·031	9·197
10–45	11·976	10·851	9·900	9·088
11–46	11·756	10·697	9·774	8·962
12–47	11·525	10·481	9·592	8·827
13–48	11·288	10·284	9·425	8·686
14–49	11·045	10·080	9·252	8·538
15–50	10·799	9·872	9·076	8·386
16–51	10·554	9·665	8·899	8·234
17–52	10·313	9·461	8·724	8·083
18–53	10·076	9·260	8·552	7·934
19–54	9·845	9·063	8·383	7·788
20–55	9·617	8·869	8·216	7·643
21–56	9·394	8·679	8·053	7·502
22–57	9·174	8·491	7·891	7·362
23–58	8·951	8·299	7·725	7·218
24–59	8·725	8·104	7·556	7·070
25–60	8·495	7·906	7·383	6·919
26–61	8·263	7·704	7·207	6·764
27–62	8·028	7·499	7·027	6·605
28–63	7·785	7·286	6·839	6·439
29–64	7·539	7·069	6·648	6·268
30–65	7·286	6·844	6·447	6·089

TABLE XXXV Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–66	7·028	6·615	6·243	5·905
32–67	6·768	6·382	6·033	5·717
33–68	6·504	6·146	5·820	5·524
34–69	6·239	5·906	5·603	5·326
35–70	5·971	5·663	5·382	5·125
36–71	5·703	5·419	5·159	4·920
37–72	5·435	5·174	4·934	4·714
38–73	5·169	4·930	4·710	4·507
39–74	4·908	4·690	4·488	4·301
40–75	4·656	4·457	4·272	4·101
41–76	4·420	4·238	4·069	3·912
42–77	4·184	4·019	3·865	3·722
43–78	3·942	3·794	3·655	3·525
44–79	3·685	3·552	3·428	3·312
45–80	3·426	3·308	3·197	3·098
46–81	3·176	3·072	2·973	2·881
47–82	2·936	2·843	2·756	2·673
48–83	2·714	2·632	2·554	2·481
49–84	2·544	2·470	2·400	2·334
50–85	2·388	2·322	2·258	2·198
51–86	2·248	2·188	2·131	2·077
52–87	2·117	2·063	2·012	1·963
53–88	2·068	1·960	1·914	1·870
54–89	1·858	1·817	1·778	1·740
55–90	1·666	1·633	1·601	1·570
56–91	1·402	1·377	1·353	1·330
57–92	1·120	1·102	1·085	1·069
58–93	·794	·784	·773	·763
59–94	·511	·505	·499	·494
60–95	·233	·230	·228	·226
61–96	·000	·000	·000	·000

TABLE XXXVI.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton*.

Difference of age 40 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–41	9·523	8·585	7·800	7·135
2–42	10·907	9·639	8·942	8·182
3–43	11·343	10·242	9·315	8·528
4–44	11·578	10·468	9·531	8·733
5–45	11·597	10·500	9·571	8·778
6–46	11·610	10·528	9·609	8·823
7–47	11·550	10·491	9·589	8·815
8–48	11·435	10·404	9·524	8·767
9–49	11·260	10·263	9·409	8·673
10–50	11·044	10·085	9·260	8·548
11–51	10·816	9·894	9·100	8·411
12–52	10·582	9·698	8·934	8·270
13–53	10·344	9·497	8·763	8·123
14–54	10·100	9·290	8·586	7·970
15–55	9·851	9·077	8·403	7·812
16–56	9·595	8·858	8·214	7·648
17–57	9·340	8·639	8·024	7·481
18–58	9·089	8·422	7·835	7·316
19–59	8·841	8·207	7·648	7·153
20–60	8·597	7·995	7·463	6·990
21–61	8·357	7·787	7·281	6·830
22–62	8·119	7·580	7·100	6·670
23–63	7·874	7·365	6·910	6·503
24–64	7·626	7·147	6·717	6·331
25–65	7·370	6·920	6·515	6·151
26–66	7·110	6·689	6·309	5·966
27–67	6·847	6·454	6·098	5·776
28–68	6·581	6·215	5·883	5·581
29–69	6·313	5·973	5·664	5·383
30–70	6·043	5·729	5·442	5·180

TABLE XXXVI Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–71	5·772	5·483	5·218	4·974
32–72	5·502	5·236	4·992	4·767
33–73	5·235	4·991	4·766	4·559
34–74	4·973	4·749	4·543	4·353
35–75	4·720	4·516	4·327	4·152
36–76	4·481	4·295	4·123	3·962
37–77	4·242	4·073	3·916	3·770
38–78	3·996	3·844	3·702	3·570
39–79	3·734	3·598	3·471	3·352
40–80	3·469	3·349	3·236	3·130
41–81	3·216	3·109	3·009	2·914
42–82	2·973	2·878	2·789	2·705
43–83	2·750	2·666	2·587	2·511
44–84	2·581	2·505	2·433	2·365
45–85	2·424	2·356	2·291	2·230
46–86	2·282	2·221	2·162	2·107
47–87	2·148	2·093	2·041	1·991
48–88	2·036	1·987	1·941	1·895
49–89	1·882	1·840	1·800	1·761
50–90	1·685	1·651	1·619	1·590
51–91	1·417	1·391	1·367	1·343
52–92	1·130	1·113	1·095	1·079
53–93	·801	·790	·780	·770
54–94	·515	·509	·503	·498
55–95	·234	·232	·230	·228
56–96	·000	·000	·000	·000

TABLE XXXVII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 45 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–46	8·888	8·071	7·379	6·787
2–47	10·147	9·221	8·435	7·760
3–48	10·515	9·566	8·759	8·063
4–49	10·697	9·744	8·932	8·230
5–50	10·679	9·742	8·941	8·248
6–51	10·664	9·745	8·956	8·271
7–52	10·586	9·690	8·919	8·248
8–53	10·458	9·591	8·841	8·188
9–54	10·276	9·442	8·718	8·085
10–55	10·055	9·256	8·560	7·951
11–56	9·814	9·052	8·386	7·801
12–57	9·566	8·839	8·203	7·643
13–58	9·312	8·622	8·015	7·479
14–59	9·053	8·399	7·821	7·310
15–60	8·790	8·170	7·622	7·135
16–61	8·521	7·935	7·416	6·953
17–62	8·252	7·700	7·208	6·770
18–63	7·981	7·462	6·998	6·583
19–64	7·714	7·226	6·789	6·396
20–65	7·444	6·986	6·576	6·205
21–66	7·177	6·749	6·364	6·015
22–67	6·911	6·512	6·151	5·824
23–68	6·643	6·271	5·934	5·628
24–69	6·372	6·027	5·713	5·427
25–70	6·099	5·780	5·489	5·223
26–71	5·826	5·532	5·263	5·016
27–72	5·554	5·283	5·035	4·807
28–73	5·284	5·036	4·808	4·597
29–74	5·019	4·792	4·583	4·390
30–75	4·764	4·557	4·365	4·188

TABLE XXXVII Continued.

Ages	3 per Cent.	4 per Cent	5 pet Cent	6 per Cent
31–76	4·523	4·335	4·160	3·997
32–77	4·282	4·111	3·952	3·804
33–78	4·035	3·881	3·737	3·602
34–79	3·771	3·633	3·505	3·384
35–80	3·506	3·383	3·268	3·160
36–81	3·251	3·142	3·040	2·944
37–82	3·005	2·909	2·818	2·733
38–83	2·779	2·694	2·613	2·537
39–84	2·607	2·530	2·457	2·388
40–85	2·448	2·379	2·313	2·251
41–86	2·304	2·241	2·182	2·126
42–87	2·168	2·113	2·060	2·009
43–88	2·055	2·006	1·959	1·914
44–89	1·901	1·859	1·818	1·779
45–90	1·702	1·668	1·635	1·604
46–91	1·431	1·405	1·380	1·356
47–92	1·140	1·122	1·105	1·089
48–93	·808	·797	·786	·776
49–94	·519	·512	·507	·501
50–95	·235	·233	·231	·229
51–96	·000	·000	·000	·000

TABLE XXXVIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 50 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–51	8·171	7·479	6·885	6·370
2–52	9·300	8·520	7·848	7·264
3–53	9·611	8·815	8·128	7·529
4–54	9·751	8·957	8·269	7·668
5–55	9·707	8·931	8·256	7·665
6–56	9·659	8·902	8·241	7·662
7–57	9·549	8·817	8·176	7·612
8–58	9·395	8·691	8·073	7·527
9–59	9·191	8·519	7·927	7·403
10–60	8·952	8·314	7·750	7·250
11–61	8·696	8·092	7·557	7·081
12–62	8·433	7·863	7·357	6·905
13–63	8·161	7·625	7·147	6·719
14–64	7·884	7·381	6·931	6·527
15–65	7·597	7·127	6·705	6·325
16–66	7·304	6·866	6·472	6·115
17–67	7·012	6·604	6·236	5·903
18–68	6·721	6·343	6·001	5·689
19–69	6·434	6·084	5·766	5·476
20–70	6·149	5·826	5·532	5·262
21–71	5·870	5·572	5·300	5·050
22–72	5·595	5·321	5·070	4·840
23–73	5·323	5·072	4·841	4·628
24–74	5·056	4·827	4·615	4·419
25–75	4·799	4·589	4·396	4·216
26–76	4·556	4·365	4·188	4·024
27–77	4·313	4·140	3·979	3·829
28–78	4·064	3·908	3·762	3·626
29–79	3·798	3·659	3·528	3·406
30–80	3·530	3·406	3·290	3·181

TABLE XXXVIII Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–81	3·274	3·164	3·060	2·963
32–82	3·027	2·929	2·838	2·751
33–83	2·800	2·713	2·632	2·555
34–84	2·627	2·549	2·476	2·406
35–85	2·468	2·398	2·331	2·268
36–86	2·323	2·260	2·200	2·143
37–87	2·187	2·130	2·077	2·026
38–88	2·072	2·022	1·974	1·929
39–89	1·915	1·872	1·832	1·792
40–90	1·713	1·679	1·646	1·614
41–91	1·439	1·413	1·388	1·364
42–92	1·146	1·128	1·111	1·094
43–93	·811	·800	·790	·779
44–94	·521	·515	·509	·503
45–95	·236	·234	·232	·230
46–96	·000	·000	·000	·000

TABLE XXXIX.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age *55* years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–56	7·412	6·843	6·346	5·911
2–57	8·392	7·756	7·199	6·709
3–58	8·630	7·986	7·421	6·922
4–59	8·712	8·075	7·514	7·017
5–60	8·629	8·011	7·466	6·982
6–61	8·542	7·944	7·415	6·945
7–62	8·400	7·828	7·319	6·865
8–63	8·214	7·669	7·184	6·750
9–64	7·984	7·470	7·010	6·598
10–65	7·718	7·236	6·803	6·414
11–66	7·437	6·987	6·581	6·215
12–67	7·149	6·730	6·351	6·009
13–68	6·857	6·468	6·116	5·796
14–69	6·562	6·202	5·876	5·578
15–70	6·264	5·933	5·631	5·355
16–71	5·964	5·660	5·382	5·127
17–72	5·667	5·389	5·133	4·899
18–73	5·378	5·123	4·889	4·673
19–74	5·098	4·866	4·651	4·453
20–75	4·831	4·619	4·424	4·242
21–76	4·583	4·391	4·212	4·046
22–77	4·339	4·164	4·001	3·850
23–78	4·087	3·930	3·783	3·646
24–79	3·820	3·679	3·548	3·424
25–80	3·550	3·425	3·308	3·198
26–81	3·292	3·181	3·077	2·979
27–82	3·043	2·945	2·853	2·765
28–83	2·815	2·728	2·646	2·568
29–84	2·641	2·563	2·489	2·418
30–85	2·481	2·411	2·344	2·280

TABLE XXXIX Continued.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
31–86	2·336	2·272	2·212	2·154
32–87	2·108	2·142	2·088	2·036
33–88	2·083	2·033	1·985	1·939
34–89	1·925	1·882	1·841	1·802
35–90	1·723	1·688	1·654	1·622
36–91	1·446	1·420	1·395	1·371
37–92	1·152	1·134	1·116	1·099
38–93	·815	·804	·793	·783
39–94	·523	·517	·511	·505
40–95	·237	·235	·233	·231
41–96	·000	·000	·000	·000

TABLE XL.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 60 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–61	6·571	6·123	5·725	5·372
2–62	7·391	6·894	6·452	6·059
3–63	7·545	7·048	6·605	6·209
4–64	7·562	7·076	6·641	6·251
5–65	7·429	6·963	6·546	6·171
6–66	7·290	6·846	6·447	6·087
7–67	7·104	6·684	6·306	5·963
8–68	6·884	6·490	6·134	5·811
9–69	6·628	6·262	5·929	5·626
10–70	6·347	6·008	5·700	5·418
11–71	6·056	5·744	5·460	5·199
12–72	5·763	5·478	5·216	4·976
13–73	5·473	5·212	4·972	4·751
14–74	5·188	4·950	4·731	4·528
15–75	4·911	4·695	4·495	4·310
16–76	4·649	4·452	4·270	4·101
17–77	4·388	4·210	4·045	3·892
18–78	4·123	3·964	3·815	3·677
19–79	3·846	3·704	3·571	3·447
20–80	3·569	3·443	3·325	3·214
21–81	3·307	3·195	3·091	2·992
22–82	3·057	2·958	2·865	2·777
23–83	2·828	2·740	2·657	2·579
24–84	2·653	2·574	2·499	2·429
25–85	2·492	2·421	2·354	2·290
26–86	2·346	2·282	2·221	2·163
27–87	2·208	2·151	2·096	2·044
28–88	2·091	2·041	1·992	1·946
29–89	1·933	1·889	1·848	1·808
30–90	1·729	1·694	1·660	1·628
31–91	1·451	1·425	1·400	1·376
32–92	1·155	1·137	1·119	1·102
33–93	·817	·806	·795	·785
34–94	·524	·518	·512	·506
35–95	·238	·235	·233	·231

TABLE XLI.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton.*

Difference of age 65 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–66	5·633	5·295	4·996	4·728
2–67	6·266	5·896	5·569	5·276
3–68	6·330	5·965	5·641	5·352
4–69	6·277	5·924	5·611	5·332
5–70	6·102	5·768	5·472	5·209
6–71	5·925	5·610	5·331	5·084
7–72	5·714	5·418	5·157	4·929
8–73	5·480	5·204	4·963	4·752
9–74	5·225	4·969	4·747	4·556
10–75	4·962	4·725	4·522	4·350
11–76	4·707	4·487	4·301	4·148
12–77	4·449	4·368	4·195	3·943
13–78	4·185	4·022	3·871	3·729
14–79	3·904	3·759	3·624	3·497
15–80	3·621	3·492	3·372	3·259
16–81	3·348	3·235	3·128	3·028
17–82	3·087	2·987	2·893	2·804
18–83	2·849	2·760	2·677	2·598
19–84	2·668	2·589	2·513	2·442
20–85	2·503	2·431	2·364	2·299
21–86	2·354	2·290	2·229	2·171
22–87	2·216	2·158	2·104	2·051
23–88	2·099	2·048	1·999	1·953
24–89	1·939	1·895	1·854	1·814
25–90	1·734	1·699	1·665	1·633
26–91	1·455	1·429	1·404	1·379
27–92	1·158	1·140	1·122	1·105
28–93	·819	·808	·797	·786
29–94	·525	·519	·513	·507
30–95	·238	·236	·234	·231
31–96	·000	·000	·000	·000

TABLE XLII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made at *Northampton*.

Difference of age 70 years.

Ages	3 per Cent	4 per Cent	5 per Cent	6 per Cent
1–71	4·611	4·380	4·169	3·976
2–72	5·061	4·814	4·588	4·380
3–73	5·051	4·811	4·591	4·389
4–74	4·953	4·726	4·516	4·323
5–75	4·768	4·557	4·362	4·181
6–76	4·599	4·403	4·221	4·053
7–77	4·402	4·222	4·055	3·899
8–78	4·180	4·016	3·864	3·722
9–79	3·921	3·775	3·638	3·510
10–80	3·647	3·517	3·395	3·281
11–81	3·380	3·264	3·156	3·054
12–82	3·122	3·020	2·924	2·833
13–83	2·884	2·794	2·709	2·628
14–84	2·703	2·622	2·545	2·472
15–85	2·535	2·462	2·393	2·327
16–86	2·380	2·315	2·253	2·194
17–87	2·235	2·177	2·121	2·069
18–88	2·112	2·061	2·012	1·965
19–89	1·948	1·904	1·862	1·822
20–90	1·739	1·704	1·670	1·638
21–91	1·459	1·432	1·407	1·382
22–92	1·160	1·142	1·124	1·107
23–93	·820	·809	·798	·788
24–94	·526	·520	·514	·508
25–95	·238	·236	·234	·232
26–96	·000	·000	·000	·000

TABLE XLIII.

Showing the value of an annuity on *Three equal* Joint Lives, deduced from the observations made at *Northampton.*

Common Age	4 per Cent	Common Age	4 per Cent	Common Age	4 per Cent
1	5·309	36	8·448	71	2·810
2	8·251	37	8·309	72	2·627
3	9·632	38	8·165	73	2·448
4	10·661	39	8·017	74	2·277
5	11·170	40	7·865	75	2·119
6	11·707	41	7·714	76	1·985
7	12·058	42	7·567	77	1·855
8	12·266	43	7·423	78	1·720
9	12·298	44	7·276	79	1·563
10	12·200	45	7·126	80	1·400
11	12·043	46	6·972	81	1·245
12	11·865	47	6·813	82	1·092
13	11·678	48	6·650	83	·949
14	11·481	49	6·482	84	·860
15	11·274	50	6·317	85	·782
16	11·056	51	6·161	86	·716
17	10·845	52	6·011	87	·662
18	10·656	53	5·859	88	·646
19	10·490	54	5·705	89	·614
20	10·342	55	5·550	90	·563
21	10·222	56	5·393	91	·452
22	10·118	57	5·235	92	·337
23	10·012	58	5·076	93	·185
24	9·905	59	4·916	94	·085
25	9·796	60	4·755	95	·015
26	9·685	61	4·593		
27	9·572	62	4·432		
28	9·457	63	4·263		
29	9·340	64	4·093		
30	9·221	65	3·914		
31	9·099	66	3·733		
32	8·975	67	3·550		
33	8·848	68	3·366		
34	8·718	69	3·181		
35	8·585	70	2·995		

TABLE XLIV.

Showing the value of an annuity on *Three* Joint Lives, whose differences of age are 10 and 20 years, deduced from the observations made at *Northampton*.

Ages	4 per Cent	Ages	4 per Cent
1–11–21	8·627	41–51–61	5·827
2–12–22	9·914	42–52–62	5·662
3–13–23	10·344	43–53–63	5·494
4–14–24	10·598	44–54–64	5·322
5–15–25	10·655	45–55–65	5·145
6–16–26	10·708	46–56–66	4·965
7–17–27	10·700	47–57–67	4·782
8–18–28	10·654	48–58–68	4·597
9–19–29	10·562	49–59–69	4·408
10–20–30	10·438	50–60–70	4·219
11–21–31	10·305	51–61–71	4·032
12–22–32	10·170	52–62–72	3·847
13–23–33	10·031	53–63–73	3·660
14–24–34	9·887	54–64–74	3·477
15–25–35	9·738	55–65–75	3·298
16–26–36	9·584	56–66–76	3·128
17–27–37	9·429	57–67–77	2·959
18–28–38	9·278	58–68–78	2·785
19–29–39	9·131	59–69–79	2·598
20–30–40	8·986	60–70–80	2·408
21–31–41	8·850	61–71–81	2·224
22–32–42	8·718	62–72–82	2·044
23–33–43	8·586	63–73–83	1·875
24–34–44	8·451	64–74–84	1·743
25–35–45	8·313	65–75–85	1·623
26–36–46	8·171	66–76–86	1·519
27–37–47	8·027	67–77–87	1·425
28–38–48	7·878	68–78–88	1·350
29–39–49	7·725	69–79–89	1·248
30–40–50	7·571	70–80–90	1·122
31–41–51	7·420	71–81–91	·951
32–42–52	7·272	72–82–92	·767
33–43–53	7·123	73–83–93	·548
34–44–54	6·971	74–84–94	·362
35–45–55	6·816	75–85–95	·169
36–46–56	6·658		
37–47–57	6·497		
38–48–58	6·332		
39–49–59	6·164		
40–50–60	5·994		

TABLE XLV.

Showing the number of persons *living* and *dying* at every age, according to the observations made in *London*.

Age	Living	Dying	Age	Living	Dying	Age	Living	Dying
0	1000	320						
1	680	133	31	294	7	61	97	5
2	547	51	32	287	7	62	92	5
3	496	27	33	280	7	63	87	5
4	469	17	34	273	7	64	82	5
5	452	12	35	266	7	65	77	5
6	440	10	36	259	7	66	72	5
7	430	8	37	252	7	67	67	5
8	422	7	38	245	8	68	62	4
9	415	5	39	237	8	69	58	4
10	410	5	40	229	7	70	54	4
11	405	5	41	222	8	71	50	4
12	400	5	42	214	8	72	46	4
13	395	5	43	206	7	73	42	3
14	390	5	44	199	7	74	39	3
15	385	5	45	192	7	75	36	3
16	380	5	46	185	7	76	33	3
17	375	5	47	178	7	77	30	3
18	370	5	48	171	6	78	27	2
19	365	5	49	165	6	79	25	
20	360	5	50	159	6			
21	355	5	51	153	6			
22	350	5	52	147	6			
23	345	6	53	141	6			
24	339	6	54	135	6			
25	333	6	55	129	6			
26	327	6	56	123	6			
27	321	6	57	117	5			
28	315	7	58	112	5			
29	308	7	59	107	5			
30	301	7	60	102	5			

TABLE XLVI.

Showing the *Expectation of Life*, deduced from the observations made in *London*.

Age	Expectation	Age	Expectation	Age	Expectation
0	19·2				
1	27·0	31	23·1	61	12·0
2	32·0	32	22·7	62	11·6
3	34·0	33	22·3	63	11·2
4	35·6	34	21·9	64	10·8
5	36·0	35	21·5	65	10·5
6	36·0	36	21·1	66	10·1
7	35·8	37	20·7	67	9·8
8	35·6	38	20·3	68	9·4
9	35·2	39	19·9	69	9·1
10	34·8	40	19·6	70	8·8
11	34·3	41	19·2	71	8·4
12	33·7	42	18·8	72	8·1
13	33·1	43	18·5	73	7·8
14	32·5	44	18·1	74	7·5
15	31·9	45	17·8	75	7·2
16	31·3	46	17·4	76	6·8
17	30·7	47	17·0	77	6·4
18	30·1	48	16·7	78	6·0
19	29·5	49	16·3	79	5·5
20	28·9	50	16·0	80	5·0
21	28·3	51	15·6		
22	27·7	52	15·2		
23	27·2	53	14·9		
24	26·6	54	14·5		
25	26·1	55	14·2		
26	25·6	56	13·8		
27	25·1	57	13·4		
28	24·6	58	13·1		
29	24·1	59	12·7		
30	23·6	60	12·4		

TABLE XLVII.

Showing the value of an annuity on a Single Life, at every age, deduced from the observations made in *London*.

Age	3 per Cent	4 per Cent	5 per Cent	Age	3 per Cent	4 per Cent	5 per Cent	Age	3 per Cent	4 per Cent	5 per Cent
6	18·8	16·2	14·1	31	14·8	12·9	11·4	56	10·1	9·1	8·4
7	18·9	16·3	14·2	32	14·6	12·7	11·3	57	9·9	8·9	8·2
8	19·0	16·4	14·3	33	14·4	12·6	11·2	58	9·6	8·7	8·1
9	19.0	16·4	14·3	34	14·2	12·4	11·0	59	9·4	8·6	8·0
10	19·0	16·4	14·3	35	14·1	12·3	10·9	60	9·2	8·4	7·9
11	19·0	16·4	14·3	36	13·9	12·1	10·8	61	8·9	8·2	7·7
12	18·9	16·3	14·2	37	13·7	11·9	10·6	62	8·7	8·1	7·6
13	18·7	16·2	14·1	38	13·5	11·8	10·5	63	8·5	7·9	7·4
14	18·5	16·0	14·0	39	13·3	11·6	10·4	64	8·3	7·7	7·3
15	18·3	15·8	13·9	40	13·2	11·5	10·3	65	8·0	7·5	7·1
16	18·1	15·6	13·7	41	13·0	11·4	10·2	66	7·8	7·3	6·9
17	17·9	15·4	13·5	42	12·8	11·2	10·1	67	7·6	7·1	6·7
18	17·6	15·2	13·4	43	12·6	11·1	10·0	68	7·4	6·9	6·6
19	17·4	15·0	13·2	44	12·5	11·0	9·9	69	7·1	6·7	6·4
20	17·2	14·8	13·0	45	12·3	10·8	9·8	70	6·9	6·5	6·2
21	17·0	14·7	12·9	46	12·1	10·7	9·7	71	6·7	6·3	6·0
22	16·8	14·5	12·7	47	11·9	10·5	9·5	72	6·5	6·1	5·8
23	16·5	14·3	12·6	48	11·8	10·4	9·4	73	6·2	5·9	5·6
24	16·3	14·1	12·4	49	11·6	10·2	9·3	74	5·9	5·6	5·4
25	16·1	14·0	12·3	50	11·4	10·1	9·2	75	5·6	5·4	5·2
26	15·9	13·8	12·1	51	11·2	9·9	9·0				
27	15·6	13·6	12·0	52	11·0	9·8	8·9				
28	15·4	13·4	11·8	53	10·7	9·6	8·8				
29	15·2	13·2	11·7	54	10·5	9·4	8·6				
30	15·0	13·1	11·6	55	10·3	9·3	8·5				

TABLE XLVIII.

Showing the value of an annuity on Two Joint Lives, deduced from the observations made in *London*.

Ages		3 per Cent	4 per Cent	5 per Cent
10	10	14·7	13·0	11·6
	15	14·3	12·7	11·3
	20	13·8	12·2	10·8
	25	13·1	11·6	10·2
	30	12·3	10·9	9·7
	35	11·5	10·2	9·1
	40	10·7	9·6	8·6
	45	10·0	9·0	8·1
	50	9·3	8·4	7·6
	55	8·6	7·8	7·1
	60	7·8	7·2	6·6
	65	6·9	6·5	6·1
	70	6·1	5·8	5·5
	75	5·3	5·1	4·9
15	15	13·9	12·3	11·0
	20	13·3	11·8	10·5
	25	12·6	11·2	10·1
	30	11·9	10·6	9·5
	35	11·2	10·0	9·0
	40	10·4	9·4	8·5
	45	9·6	8·8	8·0
	50	8·9	8·2	7·5
	55	8·2	7·6	7·0
	60	7·5	7·0	6·5
	65	6·8	6·4	6·0
	70	6·0	5·7	5·4
	75	5·2	5·0	4·8
20	20	12·8	11·3	10·1
	25	12·2	10·8	9·7
	30	11·6	10·3	9·2

Ages		3 per Cent	4 per Cent	5 per Cent
20	35	10·9	9·8	8·8
	40	10·2	9·2	8·4
	45	9·5	8·6	7·9
	50	8·8	8·0	7·4
	55	8·1	7·5	6·9
	60	7·4	6·9	6·4
	65	6·7	6·3	5·9
	70	6·0	5·7	5·4
	75	5·2	5·0	4·8
25	25	11·8	10·5	9·4
	30	11·3	10·1	9·0
	35	10·7	9·6	8·6
	40	10·0	9·1	8·2
	45	9·4	8·5	7·8
	50	8·7	7·9	7·3
	55	8·0	7·4	6·8
	60	7·3	6·8	6·3
	65	6·6	6·2	5·8
	70	5·9	5·6	5·3
	75	5·1	4·9	4·7
30	30	10·8	9·6	8·6
	35	10·3	9·2	8·3
	40	9·7	8·8	8·0
	45	9·1	8·3	7·6
	50	8·5	7·8	7·2
	55	7·9	7·3	6·7
	60	7·2	6·7	6·2
	65	6·5	6·1	5·7
	70	5·8	5·5	5·2
	75	5·1	4·9	4·7

TABLE XLVIII Continued.

Ages		3 per Cent	4 per Cent	5 per Cent	Ages		3 per Cent	4 per Cent	5 per Cent
35	35	9·9	8·8	8·0	50	50	7·6	6·8	6·2
	40	9·4	8·5	7·7		55	7·2	6·5	6·0
	45	8·9	8·1	7·4		60	6·7	6·1	5·7
	50	8·3	7·6	7·0		65	6·2	5·7	5·3
	55	7·7	7·1	6·6		70	5·5	5·2	4·9
	60	7·1	6·5	6·1		75	4·8	4·6	4·4
	65	6·4	6·0	5·6	55	55	6·9	6·2	5·7
	70	5·7	5·4	5·1		60	6·5	5·9	5·5
	75	5·0	4·8	4·6		65	6·0	5·6	5·2
40	40	9·1	8·1	7·3		70	5·4	5·1	4·8
	45	8·7	7·8	7·1		75	4·7	4·5	4·3
	50	8·2	7·4	6·8	60	60	6·1	5·6	5·2
	55	7·6	6·9	6·4		65	5·7	5·3	4·9
	60	7·0	6·4	6·0		70	5·2	4·9	4·6
	65	6·4	5·9	5·5		75	4·6	4·4	4·2
	70	5·7	5·4	5·1	65	65	5·4	5·0	4·7
	75	5·0	4·8	4·6		70	4·9	4·6	4·4
45	45	8·3	7·4	6·7		75	4·4	4·2	4·0
	50	7·9	7·1	6·5	70	70	4·6	4·4	4·2
	55	7·4	6·7	6·2		75	4·2	4·0	3·9
	60	6·8	6·3	5·8	75	75	3·8	3·7	3·6
	65	6·3	5·8	5·4					
	70	5·6	5·3	5·0					
	75	4·9	4·7	4·5					

TABLE XLIX.

Showing the value of an annuity on a Single Life, according to the hypothesis of *M. De Moivre.*

Age	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
8	19·736	18·160	16·791	15·595	14·544	12·790
9	19·868	18·269	16·882	15·672	14·607	12·839
10	19·868	18·269	16·882	15·672	14·607	12·839
11	19·736	18·160	16·791	15·595	14·544	12·790
12	19·604	18·049	16·698	15·517	14·480	12·741
13	19·469	17·937	16·604	15·437	14·412	12·691
14	19·331	17·823	16·508	15·356	14·342	12·639
15	19·192	17·707	16·410	15·273	14·271	12·586
16	19·050	17·588	16·311	15·189	14·197	12·532
17	18·905	17·467	16·209	15·102	14·123	12·476
18	18·759	17·344	16·105	15·015	14·047	12·419
19	18·610	17·220	15·999	14·923	13·970	12·361
20	18·458	17·093	15·891	14·831	13·891	12·301
21	18·305	16·963	15·781	14·737	13·810	12·239
22	18·148	16·830	15·669	14·641	13·727	12·177
23	17·990	16·696	15·554	14·543	13·642	12·112
24	17·827	16·559	15·437	14·442	13·555	12·045
25	17·664	16·419	15·318	14·340	13·466	11·978
26	17·497	16·277	15·197	14·235	13·375	11·908
27	17·327	16·133	15·073	14·128	13·282	11·837
28	17·154	15·985	14·946	14·018	13·186	11·763
29	16·979	15·835	14·816	13·905	13·088	11·688
30	16·800	15·682	14·684	13·791	12·988	11·610
31	16·620	15·526	14·549	13·673	12·885	11·530
32	16·436	15·367	14·411	13·553	12·780	11·449
33	16·248	15·204	14·270	13·430	12·673	11·365
34	16·057	15·039	14·126	13·304	12·562	11·278
35	15·864	14·871	13·979	13·175	12·449	11·189
36	15·666	14·699	13·829	13·044	12·333	11·098
37	15·465	14·524	13·676	12·909	12·214	11·003
38	15·260	14·345	13·519	12·771	12·091	10·907
39	15·053	14·163	13·359	12·630	11·966	10·807
40	14·842	13·978	13·196	12·485	11·837	10·704
41	14·626	13·789	13·028	12·337	11·705	10·599
42	14·407	13·596	12·858	12·185	11·570	10·490
43	14·185	13·399	12·683	12·029	11·431	10·378
44	13·958	13·199	12·504	11·870	11·288	10·263
45	13·728	12·993	12·322	11·707	11·142	10·144

TABLE XLIX Continued.

Age	3 per Cent	3½ per Cent	4 per Cent	4½ per Cent	5 per Cent	6 per Cent
46	13·493	12·784	12·135	11·540	10·992	10·021
47	13·254	12·571	11·944	11·368	10·837	9·895
48	13·012	12·354	11·748	11·192	10·679	9·765
49	12·764	12·131	11·548	11·012	10·515	9·630
50	12·511	11·904	11·344	10·827	10·348	9·492
51	12·255	11·673	11·135	10·638	10·176	9·349
52	11·994	11·437	10·921	10·443	9·999	9·201
53	11·729	11·195	10·702	10·243	9·817	9·049
54	11·457	10·950	10·478	10·039	9·630	8·891
55	11·183	10·698	10·248	9·829	9·437	8·729
56	10·902	10·443	10·014	9·614	9·239	8·561
57	10·616	10·181	9·773	9·393	9·036	8·387
58	10·325	9·913	9·527	9·166	8·826	8·208
59	10·029	9·640	9·275	8·933	8·611	8·023
60	9·727	9·361	9·017	8·694	8·389	7·831
61	9·419	9·076	8·753	8·449	8·161	7·633
62	9·107	8·786	8·482	8·197	7·926	7·428
63	8·787	8·488	8·205	7·938	7·684	7·216
64	8·462	8·185	7·921	7·672	7·435	6·997
65	8·132	7·875	7·631	7·399	7·179	6·770
66	7·794	7·558	7·333	7·119	6·915	6·535
67	7·450	7·234	7·027	6·831	6·643	6·292
68	7·099	6·902	6·714	6·534	6·362	6·040
69	6·743	6·565	6·394	6·230	6·073	5·779
70	6·378	6·219	6·065	5·918	5·775	5·508
71	6·008	5·865	5·728	5·596	5·468	5·228
72	5·631	5·505	5·383	5·265	5·152	4·937
73	5·246	5·136	5·029	4·926	4·826	4·636
74	4·854	4·759	4·666	4·576	4·489	4·324
75	4·453	4·373	4·293	4·217	4·143	4·000
76	4·046	3·978	3·912	3·847	3·784	3·664
77	3·632	3·575	3·520	3·467	3·415	3·315
78	3·207	3·163	3·111	3·076	3·034	2·953
79	2·776	2·741	2·707	2·673	2·641	2·578
80	2·334	2·309	2·284	2·259	2·235	2·188
81	1·886	1·867	1·850	1·832	1·816	1·783
82	1·429	1·411	1·406	1·394	1·384	1·362
83	·961	·955	·950	·943	·937	·925
84	·484	·483	·481	·479	·476	·472
85	·000	·000	·000	·000	·000	·000

TABLE L Continued.

Ages		3 per Cent	4 per Cent	5 per Cent
45	45	9·863	9·063	8·379
	50	9·331	8·619	7·987
	55	8·662	8·044	7·500
	60	7·831	7·332	6·875
	65	6·807	6·425	6·080
	70	5·556	5·300	5·063
50	50	8·892	8·235	7·660
	55	8·312	7·738	7·230
	60	7·568	7·091	6·664
	65	6·623	6·258	5·926
	70	5·442	5·193	4·964
55	55	7·849	7·332	6·873
	60	7·220	6·781	6·386
	65	6·379	6·036	5·724
	70	5·201	5·053	4·833
60	60	6·737	6·351	6·001
	65	6·043	5·730	5·444
	70	5·081	4·858	4·653
65	65	5·547	5·277	5·031
	70	4·773	4·571	4·385
70	70	4·270	4·104	3·952

TABLE LI.

Showing the value of an Assurance of £100 on a Single Life, (either in Single or Annual Payments) deduced from the observations made at *Northampton*, and at 3 per cent interest.

This Table is adopted by *all* the Assurance Offices in London.

Age	Single Premium	Annual Premium	Age	Single Premium	Annual Premium
8 to 14	....	1·879	41	54·505	3·487
15	39·834	1·929	42	55·172	3·583
16	40·481	1·983	43	55·839	3·683
17	41·113	2·033	44	56·517	3·787
18	41·710	2·083	45	57·208	3·896
19	42·272	2·133	46	57·913	4·008
20	42·802	2·179	47	58·632	4·129
21	43·291	2·225	48	59·366	4·254
22	43·756	2·267	49	60·117	4·392
23	44·229	2·312	50	60·866	4·533
24	44·710	2·354	51	61·603	4·675
25	45·202	2·403	52	62·340	4·821
26	45·703	2·450	53	63·086	4·979
27	46·213	2·504	54	63·784	5·142
28	46·732	2·554	55	64·612	5·317
29	47·261	2·612	56	65·392	5·504
30	47·800	2·671	57	66·182	5·700
31	48·353	2·725	58	66·980	5·908
32	48·913	2·787	59	67·792	6·133
33	49·486	2·854	60	68·611	6·367
34	50·072	2·921	61	69·438	6·617
35	50·666	2·992	62	70·277	6·887
36	51·275	3·067	63	71·136	7·179
37	51·898	3·142	64	72·007	7·492
38	52·530	3·225	65	72·901	7·837
39	53·180	3·308	66	73·804	8·204
40	53·841	3·396	67	74·713	8·604

TABLE LII.

Showing the value of an assurance of £100 on Two Joint Lives, (either in Single or Annual Payments) deduced from the observations made at *Northampton*, and at 3 per cent interest.

This Table is adopted by *all* the Assurance Offices in London.

Ages		Single Premium	Annual Premium	Ages		Single Premium	Annual Premium
10	10	49·498	2·855	20	50	66·438	5·766
	15	51·177	3·053		55	69·077	6·506
	20	52·958	3·279		60	72·049	7·508
	25	54·319	3·463		65	75·406	8·930
	30	55·873	3·688	25	25	58·106	4·040
	35	57·693	3·972		30	59·322	4·248
	40	59·832	4·339		35	60·786	4·515
	45	62·206	4·794		40	62·559	4·867
	50	64·919	5·390		45	64·571	5·308
	55	67·801	6·133		50	66·923	5·893
	60	71·012	7·135		55	69·461	6·625
	65	74·606	8·557		60	72·343	7·619
15	15	52·731	3·249		65	75·621	9·035
	20	54·388	3·473	30	30	60·418	4·446
	25	55·641	3·653		35	61·754	4·708
	30	57·083	3·874		40	63·392	5·044
	35	58·783	4·154		45	65·271	5·474
	40	60·799	4·517		50	67·495	6·048
	45	63·047	4·969		55	69·915	6·769
	50	65·634	5·563		60	72·685	7·751
	55	68·395	6·303		65	75·866	9·156
	60	71·485	7·302	35	35	62·944	4·947
	65	74·960	8·719		40	64·428	5·275
20	20	55·923	3·695		45	66·149	5·692
	25	57·065	3·871		50	68·217	6·252
	30	58·390	4·087		55	70·492	6·958
	35	59·968	4·363		60	73·125	7·925
	40	61·856	4·723		65	76·181	9·316
	45	63·979	5·173				

TABLE LII Continued.

Ages		Single Premium	Annual Premium	Ages		Single Premium	Annual Premium
40	40	65·736	5·588	50	50	71·705	7·381
	45	67·274	5·988		55	73·344	8·014
	50	69·154	6·530		60	75·357	8·907
	55	71·250	7·218		65	77·831	10·226
	60	73·713	8·168	55	55	74·713	8·606
	65	76·612	9·541		60	76·443	9·451
45	45	68·611	6·367		65	78·637	10·721
	50	70·278	6·887	60	60	77·846	10·235
	55	72·164	7·551		65	79·699	11·434
	60	74·424	8·476	65	65	81·152	12·541
	65	77·134	9·825				

TABLE LIII.

Showing the value of an Assurance of £100 to be received on the decease of A, provided he dies *before* B; deduced (according to Mr. Simpson's rule) from the observations made at *Northampton*, and at 3 per cent. interest.

This Table is adopted by *all* the Assurance Offices in London; although it is one of the most *inaccurate* that ever was published. See page 511.

Age of A	Age of B	Single Premium	Annual Premium	Age of A	Age of B	Single Premium	Annual Premium
	10	24·749	1·427		10	29·461	1·824
	15	24·198	1·444		15	28·786	1·838
	20	23·498	1·455		20	27·961	1·848
	25	22·531	1·437		25	26·811	1·819
	30	21·468	1·417		30	25·546	1·788
	35	20·317	1·399		35	24·176	1·759
10	40	19·070	1·383	20	40	22·692	1·733
	45	17·696	1·364		45	21·058	1·703
	50	16·214	1·346		50	19·294	1·674
	55	14·631	1·324		55	17·410	1·640
	60	12·925	1·299		60	15·381	1·603
	65	11·098	1·273		65	13·206	1·564
	70	9·153	1·246		70	10·892	1·523
	10	26·979	1·609		10	31·789	2·027
	15	26·365	1·625		15	31·093	2·042
	20	25·602	1·635		20	30·254	2·052
	25	24·549	1·612		25	29·053	2·020
	30	23·391	1·588		30	27·683	1·982
	35	22·136	1·564		35	26·198	1·946
15	40	20·778	1·544	25	40	24·590	1·913
	45	19·281	1·520		45	22·819	1·876
	50	17·666	1·497		50	20·907	1·841
	55	15·941	1·469		55	18·866	1·799
	60	14·083	1·439		60	16·667	1·755
	65	12·092	1·407		65	14·310	1·710
	70	9·973	1·373		70	11·803	1·662

TABLE LIII Continued.

Age of A	Age of B	Single Premium	Annual Premium	Age of A	Age of B	Single Premium	Annual Premium
	10	34·404	2·271		10	44·511	3·430
	15	33·694	2·287		15	43·766	3·450
	20	32·843	2·299		20	42·921	3·471
	25	31·640	2·266		25	41·753	3·433
	30	30·209	2·223		30	40·369	3·386
	35	28·589	2·177		35	38·735	3·333
30	40	26·834	2·135	45	40	36·775	3·273
	45	24·901	2·088		45	34·306	3·183
	50	22·815	2·044		50	31·432	3·080
	55	20·588	1·993		55	28·364	2·968
	60	18·188	1·939		60	25·057	2·854
	65	15·616	1·885		65	21·514	2·740
	70	12·880	1·829		70	17·744	2·629
	10	37·375	2·573		10	48·705	4·044
	15	36·647	2·590		15	47·968	4·066
	20	35·794	2·604		20	47·144	4·091
	25	34·588	2·569		25	46·017	4·052
	30	33·166	2·526		30	44·680	4·004
	35	31·472	2·474		35	43·101	3·950
35	40	29·540	2·419	50	40	41·208	3·891
	45	27·413	2·359		45	38·846	3·807
	50	25·116	2·302		50	35·853	3·691
	55	22·664	2·237		55	32·353	3·535
	60	20·022	2·170		60	28·581	3·378
	65	17·191	2·102		65	24·540	3·224
	70	14·179	2·034		70	20·239	3·075
	10	40·763	2·956		10	53·170	4·810
	15	40·023	2·974		15	52·454	4·834
	20	39·164	2·991		20	51·668	4·867
	25	37·969	2·954		25	50·596	4·826
	30	36·560	2·909		30	49·329	4·776
	35	34·888	2·857		35	47·829	4·721
40	40	32·868	2·794	55	40	46·034	4·664
	45	30·501	2·715		45	43·800	4·583
	50	27·946	2·639		50	40·993	4·479
	55	25·218	2·555		55	37·357	4·303
	60	22·278	2·468		60	33·002	4·080
	65	19·128	2·382		65	28·336	3·863
	70	15·776	2·296		70	23·370	3·656

TABLE LIII Continued.

Age of A	Age of B	Single Premium	Annual Premium	Age of A	Age of B	Single Premium	Annual Premium
	10	58·087	5·836		10	63·510	7·285
	15	57·403	5·863		15	62·870	7·313
	20	56·669	5·905		20	62·203	7·367
	25	55·675	5·863		25	61·311	7·325
	30	54·499	5·811		30	60·251	7·271
	35	53·103	5·755		35	58·990	7·213
60	40	51·437	5·699	65	40	57·484	7·159
	45	49·367	5·622		45	55·620	7·085
	50	46·777	5·529		50	53·293	7·002
	55	43·439	5·371		55	50·302	6·858
	60	38·923	5·117		60	46·279	6·640
	65	33·419	4·795		65	40·576	6·270
	70	27·563	4·490		70	33·466	5·787

TABLE LIV.

Showing the value of an Assurance of £100 on a Single Life, according to the probabilities of living as observed by *M. De Parcieux.*

Age	4 per Cent	5 per Cent	Age	4 per Cent	5 per Cent	Age	4 per Cent	5 per Cent
6	23·550	18·843	36	34·915	28·300	66	66·573	60·838
7	23·258	18·471	37	35·558	28·886	67	67·885	62·295
8	23·092	18·233	38	36·323	29·605	68	69·158	63·714
9	22·992	18·052	39	37·119	30·357	69	70·388	65·090
10	23·046	18·033	40	37·950	31·148	70	71·562	66·405
11	23·273	18·195	41	38·819	31·981	71	72·754	67·748
12	23·677	18·543	42	39·719	32·857	72	73·869	69·024
13	24·100	18·910	43	40·665	33·781	73	74·977	70·257
14	24·538	19·290	44	41·650	34·743	74	76·069	71·500
15	24·992	19·690	45	42·677	35·776	75	77·135	72·714
16	25·462	20·110	46	43·750	36·852	76	78·262	74·010
17	25·481	20·452	47	44·781	37·886	77	79·346	75·262
18	26·281	20·814	48	45·862	38·976	78	80·365	76·438
19	26·715	21·186	49	46·896	40·024	79	81·408	77·648
20	27·162	21·576	50	47·977	41·129	80	82·323	78·710
21	27·534	21·890	51	49·019	42·190	81	83·192	79·724
22	27·923	22·210	52	50·019	43·205	82	83·985	80·643
23	28·319	22·548	53	51·058	44·271	83	84·850	81·652
24	28·731	22·890	54	52·142	45·390	84	85·846	82·838
25	29·154	23·252	55	53·181	46·467	85	86·830	83·995
26	29·592	23·624	56	54·265	47·590	86	87·750	85·086
27	30·046	24·010	57	55·396	48·776	87	88·550	86·033
28	30·515	24·414	58	56·484	49·919	88	89·573	87·262
29	31·000	24·833	59	57·615	51·121	89	90·592	88·486
30	31·500	25·271	60	58·796	52·367	90	91·588	89·686
31	32·019	25·724	61	60·027	53·686	91	92·535	90·838
32	32·558	26·195	62	61·308	55·071	92	93·415	91·890
33	33·115	26·690	63	62·562	56·429	93	94·304	92·971
34	33·692	27·205	64	63·869	57·857			
35	34·292	27·738	65	65·235	59·357			

TABLE LV.

Showing the value of an Assurance of £100 on Two Joint Lives, according to the probabilities of living as observed by *M. De Parcieux.*

Ages		3½ per Cent	4½ per Cent	Ages		3½ per Cent	4½ per Cent
10	10	37·784	30·937	30	60	65·446	58·828
	15	38·968	32·035		70	75·767	70·524
	20	40·365	33·344	35	35	49 248	41·711
	30	43·243	35·915		40	51·183	43·636
	40	47·977	40·389		45	54·108	46·667
	50	55·876	48·523		55	61·388	54·401
	60	64·665	57·941		65	70·654	64·658
	70	75·378	70 067				
15	15	40·040	33·051	40	40	52·799	45·268
	20	41·325	34·270		45	55 383	47·972
	25	42·499	35·321		50	58·643	51·452
	35	45·850	38·417		60	66 028	59·448
	45	52·123	44 622		70	75·899	70·670
	55	60·384	53·308	45	45	57·547	50·267
	65	70·201	64·138		50	60·374	53·316
20	20	42·478	35·389		55	63·394	56·580
	25	43·523	36·337		65	71·554	65·671
	30	44·883	37·577	50	50	62·714	55·878
	40	49 181	41·672		55	65 257	58 652
	50	56 712	49·449		60	68·409	62·135
	60	65·213	58·570		70	76·981	71·936
	70	75·662	70·403				
25	25	44·423	37·168	55	55	67·303	60·912
	30	45·627	38·279		60	69·941	63·854
	35	47·243	39·790		65	73·593	68·013
	45	52·988	45·548	60	60	71·990	66·166
	55	60·918	53·915		65	75·033	69·663
	65	70·522	64·512		70	78·571	72·341
30	30	46·651	39·240	65	65	77·246	73·784
	35	48·065	40·578		70	80·190	75·683
	40	50·256	42·736				
	50	57·239	50·000				

TABLE LVI.

Showing the value of an Assurance of £100 on a Single Life, according to the probabilities of living as observed in *Sweden*.

Age	Males		Females		Age	Males		Females	
	4 per Cent	5 per Cent	4 per Cent	5 per Cent		4 per Cent	5 per Cent	4 per Cent	5 per Cent
6	24·531	19·519	22·904	18·081	36	39·996	33·305	37·392	30·752
7	24·258	19·157	22·573	17·662	37	40·838	34·129	38·192	31·514
8	24·134	18·948	22·454	17·452	38	41·715	34·990	39·023	32·319
9	24·173	18·905	22·496	17·414	39	42·631	35·895	39·888	33·162
10	24·327	18·981	22·658	17·500	40	43·585	36·852	40·765	34·019
11	24·615	19·190	22·919	17·686	41	44·515	37·786	41·596	34·824
12	25·034	19·543	23·261	17·957	42	45·400	38·667	42·331	35·533
13	25·469	19·910	23·692	18·310	43	46·215	39·476	43·077	36·252
14	25·977	20·362	24·204	18·767	44	47·005	40·324	43·861	37·005
15	26·519	20·838	24·738	19·238	45	47·942	41·205	44·681	37·805
16	27·084	21·348	25·292	19·733	46	48·858	42·129	45·573	38·686
17	27·681	21·886	25·808	20·186	47	49·804	43·090	46·561	39·676
18	28·296	22·452	26·342	20·657	48	50·788	44·105	47·615	40·748
19	28·877	22·976	26·873	21·129	49	51·815	45·161	48·719	41·881
20	29·481	23·529	27·415	21·618	50	52·819	46·200	49·812	43·000
21	30·031	24·024	27·981	22·114	51	53·731	47·143	50·888	44·110
22	30·608	24·550	28·565	22·643	52	54·673	48·119	51·954	45·205
23	31·204	25·086	29·177	23·195	53	55·650	49·133	53·000	46·286
24	31·762	25·586	29·800	23·767	54	56·658	50·190	54·088	47·419
25	32·338	26·110	30·434	24·352	55	57·700	51·290	55·223	48·610
26	32·938	26·657	31·096	24·967	56	58·781	52·438	56·408	49·862
27	33·562	27·229	31·727	25·543	57	59·904	53·638	57·646	51·176
28	34·211	27·829	32·354	26·119	58	61·000	54·814	58·877	52·495
29	34·885	28·457	32·973	26·686	59	62·134	56·038	60·162	53·886
30	35·573	29·100	33·611	27·276	60	63·308	57·319	61·388	55·216
31	36·250	29·733	34·215	27·829	61	64·458	58·571	62·542	56·457
32	36·942	30·386	34·842	28·405	62	65·577	59·800	63·642	57·643
33	37·662	31·062	35·435	28·938	63	66·658	60·986	64·746	58·843
34	38·408	31·776	36·042	29·495	64	67·762	62·200	65·885	60·086
35	39·185	32·524	36·673	30·076	65	68·885	63·448	67·054	61·376

TABLE LVI Continued.

Age	Males 4 per Cent	Males 5 per Cent	Females 4 per Cent	Females 5 per Cent	Age	Males 4 per Cent	Males 5 per Cent	Females 4 per Cent	Females 5 per Cent
66	70·031	64·724	68·262	62·710	86	87·380	84·657	86·469	83·590
67	71·196	66·629	69·500	64·090	87	87·869	85·224	86·804	83·976
68	72·304	67·276	70·785	65·524	88	88·635	86·133	87·331	84·590
69	73·335	68·438	72·104	67·019	89	89·623	87·314	88·046	85·433
70	74·538	69·576	73·473	68·576	90	90·704	88·610	88·950	86·510
71	75·315	70·676	74·754	70·033	91	91·715	89·829	89·892	87·638
72	76·231	71·714	75·919	71·367	92	92·942	91·314	90·965	88·929
73	77·154	72·767	76·931	72·524	93	94·319	92·996	92·034	90·219
74	77·985	73·710	77·723	73·424	94	95·231	94·105	93·081	91·486
75	78·896	74·752	78·531	74·343	95			94·062	92·681
76	79·796	75·790	79·358	75·290	96			94·923	93·729
77	80·677	76·805	80·212	76·271					
78	81·662	77·948	81·104	77·300					
79	82·646	79·095	82·046	78·400					
80	83·615	80·229	83·069	79·596					
81	84·550	81·329	84·058	80·757					
82	85·415	82·352	84·981	81·848					
83	86·154	83·224	85·773	82·786					
84	86·642	83·795	86·311	83·429					
85	87·034	84·257	86·338	83·448					

TABLE LVII.

Showing the value of an Assurance of £100 on Two Joint Lives, according to the probabilities o living as observed in *Sweden*.

Ages		4 per Cent	Ages		4 per Cent	Ages		4 per Cent
10	10	34·073	30	54	59·950	55	55	67·327
	16	35·658		60	65·512		61	70·942
	22	37·888		66	71·227		67	75·100
	28	40·446		72	77·381		73	79·881
	34	43·304	35	35	49·581		79	84·038
	40	47·142		41	52·458		85	87·569
	46	51·177		47	55·550		91	92·008
	52	56·319		53	59·800	60	60	72·677
15	15	36·404		59	64·946		66	75·965
	21	38·458		65	70·465		72	80·196
	27	40·800		71	76·565		78	83·808
	33	43·462		77	81·412		84	87·500
	39	46·923	40	40	53·985		90	91·027
	45	50·819		46	56·592	65	65	77·381
	51	55·715		52	60·354		71	80·838
	57	61·204		58	64·908		77	83·923
20	20	39·684		64	70·142		83	87·492
	26	41·754		70	75·946		89	90·058
	32	44·185		76	80·865		95	94·400
	38	47·231		82	85·731	70	70	82·334
	44	50·958	45	45	57·869		76	84·823
	50	55·512		51	61·042		82	87·865
	56	60·662		57	64·996		88	89·765
	62	66·700		63	69·862		94	91·177
25	25	42·888		69	75·285	75	75	85·969
	31	45·062		75	80·277		81	88·396
	37	47·696		81	85·177		87	89·981
	43	51·219		87	88·142		93	93·942
	49	55·338	50	50	62·665	80	80	89·392
	55	60·204		56	65·869		86	90·708
	61	66·065		62	70·196		92	93·700
	67	72·019		68	75·050	85	85	91·492
30	30	46·288		74	79·981		91	93·365
	36	48·500		80	84·654	90	90	93·392
	42	51·758		86	87·804	95	95	96·062
	48	55·362		92	93·008			

TABLE LVIII.

Showing the present value of £1 to be received at the end of any number of years.

Years	4 per Cent	4½ per Cent	5 pet Cent	Years	4 per Cent	4½ per Cent	5 per Cent
1	·96154	·95694	·95238	31	·29646	·25550	·22036
2	·92456	·91573	·90703	32	·28506	·24450	·20987
3	·88900	·87630	·86384	33	·27409	·23397	·19987
4	·85480	·83856	·82270	34	·26355	·22390	·19035
5	·82193	·80245	·78353	35	·25342	·21425	·18129
6	·79031	·76790	·74622	36	·24367	20503	·17266
7	·75992	·73483	·71068	37	·23430	·19620	·16444
8	·73069	·70319	·67684	38	·22529	·18775	·15661
9	·70259	·67290	·64461	39	·21662	17967	·14915
10	·67556	·64393	·61391	40	·20829	·17193	·14205
11	·64958	·61620	·58468	41	·20028	·16453	·13528
12	·62460	·58966	·55684	42	·19257	·15744	·12884
13	·60057	·56427	·53032	43	·18517	·15066	·12270
14	·57748	·53997	·50507	44	·17805	·14417	·11686
15	·55526	·51672	·48102	45	·17120	·13796	·11130
16	·53391	·49447	·45811	46	·16461	·13202	·10600
17	·51337	·47318	·43630	47	·15828	·12634	·10095
18	·49363	·45280	·41552	48	·15219	·12090	·09614
19	·47464	·43330	·39573	49	·14634	·11569	·09156
20	·45639	·41464	·37689	50	·14071	·11071	·08720
21	·43883	·39679	·35894	51	·13530	·10594	·08305
22	·42196	·37970	·34185	52	·13010	·10138	·07910
23	·40573	·36335	·32557	53	·12509	·09701	·07533
24	·39012	·34770	·31007	54	·12028	·09284	·07174
25	·37512	·33273	·29530	55	·11566	·08884	·06833
26	·36069	·31840	·28124	56	·11121	·08501	06507
27	·34682	·30469	·26785	57	·10693	·08135	·06197
28	·33348	·29157	·25509	58	·10282	·07785	·05902
29	·32065	·27902	·24295	59	·09886	·07450	·05621
30	·30832	·26700	·23138	60	·09506	·07129	·05354

TABLE LVIII Continued.

Years	4 per Cent	4½ per Cent	5 per Cent	Years	4 per Cent	4½ per Cent	5 per Cent
61	·09140	·06822	·05099	81	·04172	·02829	·01922
62	·08789	·06528	·04856	82	·04011	·02707	·01830
63	·08451	·06247	·04625	83	·03857	·02590	·01743
64	·08126	·05978	·04404	84	·03709	·02479	·01660
65	·07813	·05721	·04195	85	·03566	·02372	·01581
66	·07513	·05474	·03995	86	·03429	·02270	·01506
67	·07224	·05239	·03805	87	·03297	·02172	·01434
68	·06946	·05013	·03623	88	·03170	·02079	·01366
69	·06679	·04797	·03451	89	·03048	·01989	·01301
70	·06422	·04590	·03287	90	·02931	·01903	·01239
71	·06175	·04393	·03130	91	·02818	·01821	·01180
72	·05937	·04204	·02981	92	·02710	·01743	·01124
73	·05709	·04023	·02839	93	·02606	·01668	·01070
74	·05490	·03849	·02704	94	·02505	·01596	·01019
75	·05278	·03684	·02575	95	·02409	·01527	·00971
76	·05075	·03525	·02453	96	·02316	·01462	·00924
77	·04880	·03373	·02336	97	·02227	·01399	·00880
78	·04692	·03228	·02225	98	·02142	·01338	·00838
79	·04512	·03089	·02119	99	·02059	·01281	·00798
80	·04338	·02956	·02018	100	·01980	·01226	·00760

TABLE LIX.

Showing the present value of £1 *per annum* for any number of years.

Years	4 per Cent	4½ per Cent	5 per Cent	Years	4 per Cent	4½ per Cent	5 per Cent
1	·962	·957	·952	31	17·588	16·544	15·593
2	1·886	1·873	1·859	32	17·874	16·789	15·803
3	2·775	2·749	2·723	33	18·148	17·023	16·003
4	3·630	3·588	3·546	34	18·411	17·247	16·193
5	4·452	4·390	4·329	35	18·665	17·461	16·374
6	5·242	5·158	5·076	36	18·908	17·666	16·547
7	6·002	5·893	5·786	37	19·143	17·862	16·711
8	6·733	6·596	6·463	38	19·368	18·050	16·868
9	7·435	7·269	7·108	39	19·584	18·230	17·017
10	8·111	7·913	7·722	40	19·793	18·402	17·159
11	8·760	8·529	8·306	41	19·993	18·566	17·294
12	9·385	9·119	8·863	42	20·186	18·724	17·423
13	9·986	9·683	9·394	43	20·371	18·874	17·546
14	10·563	10·223	9·899	44	20·549	19·018	17·663
15	11·118	10·740	10·380	45	20·720	19·156	17·774
16	11·652	11·234	10·838	46	20·885	19·288	17·880
17	12·166	11·707	11·274	47	21·043	19·415	17·981
18	12·659	12·160	11·690	48	21·195	19·536	18·077
19	13·134	12·593	12·085	49	21·341	19·651	18·169
20	13·590	13·008	12·462	50	21·482	19·762	18·256
21	14·029	13·405	12·821	51	21·617	19·868	18·339
22	14·451	13·784	13·163	52	21·748	19·969	18·418
23	14·857	14·148	13·489	53	21·873	20·066	18·493
24	15·247	14·495	13·799	54	21·993	20·159	18·565
25	15·622	14·828	14·094	55	22·109	20·248	18·633
26	15·983	15·147	14·375	56	22·220	20·333	18·699
27	16·330	15·451	14·643	57	22·327	20·414	18·761
28	16·663	15·743	14·898	58	22·430	20·492	18·820
29	16·984	16·022	15·141	59	22·528	20·567	18·876
30	17·292	16·289	15·372	60	22·623	20·638	18·929

TABLE LIX Continued.

Years	4 per Cent	4½ per Cent	5 per Cent	Years	4 per Cent	4½ per Cent	5 per Cent
61	22·715	20·706	18·980	86	24·143	21·718	19·699
62	22·803	20·772	19·029	87	24·170	21·740	19·713
63	22·887	20·834	19·075	88	24·207	21·760	19·727
64	22·969	20·894	19·119	89	24·238	21·780	19·740
65	23·047	20·951	19·161	90	24·267	21·799	19·752
66	23·122	21·006	19·201	91	24·295	21·817	19·764
67	23·194	21·058	19·239	92	24·323	21·835	19·775
68	23·264	21·108	19·275	93	24·349	21·852	19·786
69	23·330	21·156	19·310	94	24·374	21·868	19·796
70	23·395	21·202	19·343	95	24·398	21·883	19·806
71	23·456	21·246	19·374	96	24·421	21·897	19·815
72	23·516	21·288	19·404	97	24·443	21·911	19·824
73	23·573	21·328	19·432	98	24·465	21·925	19·832
74	23·628	21·367	19·459	99	24·485	21·938	19·840
75	23·680	21·404	19·485	100	24·505	21·950	19·848
76	23·731	21·439	19·509	Perp.	25·000	22·222	20·000
77	23·780	21·473	19·533				
78	23·827	21·505	19·555				
79	23·872	21·536	19·576				
80	23·915	21·565	19·596				
81	23·957	21·594	19·616				
82	23·997	21·621	19·634				
83	24·036	21·647	19·651				
84	24·073	21·671	19·668				
85	24·109	21·695	19·684				

THE END.

ERRATA.

Page	*line*	*for*	*read*
6	last,	§ 2	§ 27
41	17,	last	twelfth
131	11,	$(1-\phi-\chi)\times(A)^{d}-(1-\chi)\times(AB)^{d}$	$(1-\phi)\times(A)^{d}-(AB)^{d}$
144	last,	§ 197	§ 200.
153	19,	that sum	the n°. of years purchase
168	10,	$[(A+B(1-\varrho A)]$	$[A+B(1-\varrho A)]$
175	11,	$\frac{f}{A^{2}}$	$\frac{f}{A^{1}}$
240	last,	$\frac{1+ABC}{(1+\varrho)}$	$\frac{1-\varrho ABC}{(1+\varrho)}$
278	13,	$(1+\varrho)^{-n+1}$	$(1+\varrho)^{-(n+1)}$
329	5,	AA	AA
345	19,	·05	·5
352	18,	formula	formulæ
355	17,	§ 23	§ 23 and 24
380	14,	first	second
431	7,	22·234	24·107
449	7 from bottom	*dele*	by Table LVIII
514	in the first Table, 22·78	*ought to be*	22·28

In my *Doctrine of Interest and Annuities*, page 134, line 5, I have stated that the formula there deduced " is the very formula given by Dr. Halley." But this is not exactly the case, since Dr. Halley makes $\gamma=\frac{6(2\nu+n+1)+6}{n^{2}}$: and I am not aware how the difference arises. Both the formulæ, however, give the value sufficiently near for general use.

APPENDIX

TO THE

DOCTRINE

OF

LIFE-ANNUITIES AND ASSURANCES,

CONTAINING

A PAPER

Read before the Royal Society

ON

A NEW METHOD OF CALCULATING

THE VALUE

OF

LIFE-ANNUITIES.

By FRANCIS BAILY.

LONDON:
SOLD BY JOHN RICHARDSON, ROYAL EXCHANGE.

1813.

Printed by Richard Taylor and Co., Shoe-Lane, London.

PREFACE.

THE following *paper* was read before the *Royal Society* during the course of the last year: but the Committee* (to whom it is customary to refer the papers which are presented to the Society within the year, in order that they may *select* for publication such as they conceive to be not only the most *curious*, but likewise the most *useful*,) have not thought proper to publish it in their annual volume of the *Philosophical Transactions*. The *motives* which induced them to come to this decision, I shall not stop to inquire into; since such a discussion would be foreign to my purpose: but perhaps they may be evident to the reader before he closes the present work.

The importance and utility of the method which has been investigated and explained in the paper here alluded to will be apparent to every person who has

* This Committee consists of the Council for the current year; a list of whom, for the year 1812, will be found in page 20.

studied with attention the subject of Life Annuities and Assurances; and will be a sufficient apology (if any apology indeed were necessary) for my submitting it, in its present shape, to the public opinion. The want of a more numerous collection of Tables than those which we at present possess, for determining the value of Annuities on Lives, according to the Observations of human mortality at *various* places, has long been experienced and lamented. It was hoped and expected that when any person should come forward, skilful and bold enough to undertake the arduous task, he would meet with sufficient encouragement from the public to accomplish his design, and with sufficient patronage to remunerate him for the time, the labour, and the expense which must necessarily be incurred in so laudable and useful a pursuit. With still greater confidence might he hope for assistance and support from the various Life Assurance Companies that have been lately established; and who (it was natural to expect) would be desirous of obtaining all the information and help which they possibly could, on a subject in which they are so immediately interested. Nothing perhaps tended so much to destroy the numerous *Bubble* Societies which sprung up about forty years ago;—

nothing, probably, opened so effectually the eyes of the public with respect to their *delusive* schemes, as the publication of more correct and comprehensive tables of the value of Life Annuities: whereby the true value, which *ought* to be given in such cases, was more accurately determined. And, as one improvement in science generally leads on to another, this naturally opened the way to a more complete and comprehensive investigation of the subject: so that, at the present day, a new, a distinct, and an interesting branch of analysis has arisen, which was unknown to mathematicians of a former period*.

To this important circumstance, the *present* Societies (who have established their plans on a more correct, and consequently a more firm and extensive basis) are indebted not only for their origin, but also for their very existence and success at this moment. For, by the variety of tables of the value of Life Annuities which have been formed from the observations of human mortality at *different* places, they have had an opportunity of judging of the *proper* values on which the business

* It is a singular fact, that the subject of Life Annuities has never yet formed a part of any elementary treatise on mixed mathematics. Its increasing utility and importance surely may lay claim to some notice from future compilers.

of such societies ought to be conducted; and by a more enlarged analysis of the subject they have been able to facilitate their computations and extend their concerns to the more complicated cases that arise, and which formerly were not considered capable of an accurate solution. This knowledge is now more generally diffused and understood; and the public are not again likely to become the dupes of any artful impostor.

These advantages have kept pace with, and indeed are principally owing to, the gradual extension and publication of the Tables of Life Annuities; or, in other words, with our opportunities of detecting error, and of enlarging the boundaries of the science. For, the labours of the analyst in this department would have been of very little *practical* utility if they had not been supported by such tables. Those tables, however, which we at present possess, are by no means sufficiently comprehensive for all the useful purposes of life. The observations made in *Sweden* seem to be the most correct for mankind *in general;* and those given by *M. De Parcieux* appear to be peculiarly adapted for a set of *life annuitants*. But the Tables of *Life-Annuities*, which are formed from these observations,

are so limited as to be inapplicable to many cases which now arise in the ordinary course of business. On the other hand, the observations made at *Northampton* are on too contracted a scale to give the true rate of human mortality amongst mankind in general: and although their use has been continued, it is principally on account of the more numerous tables of the value of Life Annuities which have been computed therefrom. In the present advanced state of the science however, even these tables are too confined; and it would be highly desirable to possess tables of the value of annuities on two joint lives for every possible combination of age, not only according to the observations made at *Northampton*, but also according to those made in *Sweden*, and by *M. De Parcieux* in France. I now, however, despair of ever seeing this accomplished: and my reason for entertaining so discouraging an opinion arises from the want of patronage and support which an industrious and intelligent author has lately experienced from the public.

The name of this gentleman is Mr. George Barrett of Petworth in Sussex, who (at an immense expense of time and labour) has calculated and formed the most numerous and comprehensive set of Life Annuity Tables that ever were, or probably ever *will* be, produced.

The nature, the extent, and the design of those Tables will appear in the course of the following paper. It will be sufficient here to observe, that the author, flattered with the prospect of success, issued his Proposals, about eighteen months ago, for publishing them by subscription. The work was to have been comprised in two large quarto volumes; containing, in the whole, about thirteen or fourteen hundred pages closely printed. The price not to be less than four guineas each volume. In order to promote his views as far as lay in *my* power, I addressed a letter to each of the Life Assurance Companies, urging his claims to *their* patronage and support. But his hopes and expectations have been defeated in every quarter; and after an ineffectual attempt to derive assistance from the public in support of his plan, and not wishing to involve himself in any serious expenses therein, he has been reluctantly obliged to abandon his design altogether: and thus the world has been deprived of one of the most extraordinary and successful efforts of patience, perseverance and talent that ever was produced by one man. Out of all the Life Assurance Companies, now established, (fifteen in number) only two* consented to patronize the

* The Albion and The Provident Institution.

work; eleven* declined sending any answer to the application that was made; and two of them, the *Atlas* and the *Westminster* Societies, (their names *shall* be made conspicuous) absolutely *refused* to subscribe for a single copy.

It is in vain for these several Life Assurance Companies to maintain that they are already possessed of every information which they require, and of every table which may be useful to them, in their business. They must know (or rather they *ought* to know) that they are very materially interested in every improvement in the science; and that even a re-calculation (by another hand) of those tables, which they now enjoy, might serve to correct many errors which are *known* to exist in them.—Besides, it cannot be necessary to tell them that they are *not* possessed of every table requisite for carrying on their business to its fullest extent. For all these Companies are in the practice of granting assurances, on various contingencies, for *terms of years:* and I am informed that some of them, in order to abridge the necessary labour of computation, adopt a singular (but at the same time a very *inaccurate* and *unjust)* mode of determining the values in such

* The Amicable, the Eagle, the Equitable, the Globe, the Hope, the London Assurance, the London Life Assurance, the Pelican, the Rock, the Royal Exchange, and the Sun.

cases. Moreover, in the granting and purchasing of *annuities* on lives, they want *other* tables than those which are deduced from the *Northampton* observations. In determining also the value of Reversionary Sums and Annuities for *Widows*, it would be desirable to know how far the result is affected by the superior longevity of *females:* a circumstance which might be ascertained with the greatest accuracy by means of the new tables here alluded to. Indeed, upon a variety of points connected with this subject (both of curiosity and real utility) these tables were capable of throwing a very considerable degree of light and interest.—But I am ashamed to insist further on so obvious a circumstance; and I now regret that I should have so much mistaken the nature and object of these *trading* companies, as ever to have addressed them as the patrons and promoters of literature and science*.

* Dr. Price, in his account of the Equitable Society, observes that "it is indeed furnished with Tables by which a great part of "its business is conducted: but there are some important tables "which it wants, and with which it should be supplied. These, "when composed, together with all its other tables, should be "subject to the revisal and examination of the best judges, and "afterwards published; with a minute account of the princi-"ples assumed, and the method taken in composing them. Such "a publication would be a valuable addition to this part of science; "and it would also be the means of increasing and establishing "the credit of the Society." *Obs. on Rev. Pay.*, vol. i. p. 180.

Besides the Life Assurance Companies, most of the Nobility and Gentlemen possessed of landed estates, together with several of the Colleges and other Corporate Bodies, were addressed also by Mr. Barrett on this occasion. But the list of his subscribers was so small that he has not thought himself justified in sending the work to the press: and the public are thus for ever deprived of this singular monument of industry and abilities.

In order to preserve some record of Mr. Barrett's prodigious labour, as well as to explain the principles upon which his Tables are constructed—principles which open a *new* and a wide field to the analyst, and which greatly abridge the labour of computation in some of the most intricate problems that occur in the science—I drew up the following memoir, and presented it to the Royal Society, under the impression that they might think it worthy of publication in the *Philosophical Transactions*. They had, in the preceding year, given a favourable reception to a paper which I sent to them, on a subject *less deserving* their attention*; but I had then to learn, that whilst the

* On the Solar Eclipse which is said to have been predicted by Thales. See the *Phil. Trans.* for 1811, part ii., page 220.

volumes which are annually published by the Royal Society are open to such curious yet fruitless speculations, they are shut to investigations of a more serious and important nature: that whilst the dark and doubtful researches of the theorist are received with indulgence, the labours of those who are desirous of rendering themselves *practically* useful to society, and of enlarging the boundaries of *true* science, are rejected with coldness and indifference. Nevertheless, as far as my humble efforts can avail, the astonishing labours of Mr. Barrett *shall not* sink into oblivion; and I am happy in his permission to print the following paper, together with the accompanying tables, by way of *Appendix* to my *Doctrine of Life Annuities and Assurances;* whereby some record of his wonderful exertions in this particular branch of science may *still* be preserved amongst those who are interested in the subject.

I cannot omit the present opportunity which is afforded me of noticing one of the most singular instances of *perseverance in error* that perhaps ever was witnessed in the republic of letters; and, as the case is

In some measure connected with the present subject, I hope I may be excused for introducing it in this place. In the preface to my *Doctrine of Life Annuities and Assurances* (page xxiii) I have exposed the *inaccuracy* and *absurdity* of the formulæ which have been given by Mr. Morgan, in the *Philosophical Transactions*, for the solution of those cases in Contingent Assurances where three lives are concerned. That work not being within the reach of every person, Mr. Morgan has inserted the formulæ at the end of the first volume of Dr. Price's *Observations on Reversionary Payments*, Note P. But I have asserted that "the errors of the "original are multiplied in the copy: and Mr. Morgan, "if he studies his own reputation as a mathematician, "had better expunge them altogether in future, than "suffer them to remain in their present *worse than* "*useless state.*" Yet, notwithstanding all that I have there said, and although I have given ample references to the body of the work for a *demonstration* of the absurdity and errors of his formulæ, he has (within these last three months) given a new edition of Dr. Price's work above mentioned, wherein *all the erroneous parts of the former editions are faithfully retained!!!*

Mr. Morgan may perhaps flatter himself that no one will follow him through his investigations, and that he shall thereby escape the censure which he so justly deserves for this extraordinary conduct: but I shall briefly point out a case or two where the absurdity of his formulæ may be readily detected, even by those who are unacquainted with the principles from which they are deduced.

A singular instance of this kind occurs in the formula for the solution of his fourteenth problem, when the ages are equal*. For, if we take any case where the lives are under 52 years of age (the rate of interest 4 per cent, and according to the *Northampton* tables) we shall find that the value will always come out a *negative quantity !!!* It is surely unnecessary to make any comment upon so singular a result as this: and yet, if the reader is desirous of amusing himself with any further proofs of the futility of this formula, he may take any case where the lives are from 52 to 57 years of age (both inclusive); and having calculated the values according to the two rates of 4 and 5 per

* See Price's *Obs. on Rev. Pay.* vol. i. page 395. The formula there given is $\frac{S(r-1)}{r} \times (V - CC - C - CCC)$.

cent interest, he will find that all the results, according to the *former* rate, will be *positive;* but that, according to the *latter* rate, they will be still all *negative !!!* The same circumstance will occur in other instances which might be adduced.

Now, this is one of his formulæ which I had particularly noticed as erroneous *; and the remarks which I have just made are sufficient to show its *absurdity*. Indeed this case, from its simplicity, readily points out the inaccuracy of the method of solution which has been adopted; but there are *many* others, wherein the quantities involved are so numerous, so intricate, and so complex, that the absurdity of the formula cannot be shown by a practical example without employing much time and labour.

One case, however, will suffice for my present purpose. Let the reader attempt to solve the eleventh problem given by Mr. Morgan† by assuming the ages of A, B, C, to be respectively 50, 40, and 30 years, (the rate of interest 4 per cent, and according to the *Northampton* observations) and he will find that the value of an assurance of £100, payable on the con-

* In my *Doctrine of Life Annuities,* &c. page 308, note.

† See Dr. Price's *Obs. on Rev. Pay.* vol. i. page 392, case 2nd.

tingency therein mentioned, will come out equal to the sum of *six hundred and twelve pounds !!!* and there is this further remarkable absurdity attending the formula, that the *smaller* the sum to be received, the *greater* is the value of the assurance; and *vice versâ*. Thus the assurance of £1 payable on that contingency is *six hundred and ninety-five pounds;* whereas, the assurance of £800 is only *twenty-nine pounds:* and the assurance of £900 and all higher sums comes out a *negative quantity* *!!!

This is another of his formulæ which, amongst many others, I had also particularly noticed as incorrect; and I had even given the *true* quantities which (according to his own notation) *ought* to have been used instead of the erroneous ones which he has retained †. Mr. Morgan must be aware that there is a duty which he owes not only to the *public*, but also to his *employers;* and I leave it to his own sense of propriety to justify the re-publication of such *manifest errors.*

These are a *few* only of the *numerous cases* which I might adduce: but it is needless for me to insist

* I here allude to the formula in the *seventh* edition of Dr. Price's treatise; but, if the question be solved by the formula given in the *sixth* edition, the value will in *all* cases be *negative !!!*

† See *Doctrine of Life Annuities,* &c. page 274, note.

further upon this point; since I have already said enough, both here and in another place, to enable the public to draw a just and proper inference on the subject. Mr. Morgan is known to the world, as a mathematician, by those papers only (in the *Philosophical Transactions)* which he has written on *Contingent Assurances.* Those papers have been the subject of much comment in the preceding pages, and therefore I shall not add another remark upon them in this place. I wish it, however, to be understood that this branch of the science is not a subject of *curiosity* only, but one of *real utility.* Property to a very large amount (particularly in this country) is constantly depending on the contingencies mentioned in those problems to which I have referred; and it is therefore of material consequence that a proper, a correct, and a simple mode of determining the value of such property should be laid down. But every mathematician who has attended to the subject must know that *all* the formulæ, which Mr. Morgan has given for this purpose, only tend to perplex and *mislead* the public; and will not, except in very few cases, give a true and accurate (nor, in *any* case, the most simple and concise) method of solving the different questions that arise. The natural consequence

of which is, that, in the distribution of such property, and according to such rules, one party must be *defrauded;* whilst another is *unjustly* enriched at the expense of such innocent sufferer.

As the formulæ above alluded to are inserted in Dr. Price's *Observations on Reversionary Payments*, without the investigations, Mr. Morgan ought to have been more particular in seeing that they were correctly deduced and properly stated; since the reader has no opportunity of immediately detecting any error. More especially ought he to have given greater attention to this point, in any *subsequent* edition of that work, after he had been publicly attacked upon their want of accuracy. The world, however, will learn, perhaps with some surprise and indignation, that *he has not corrected a single error!!!* and that in the seventh edition of Dr. Price's treatise, which has just been published, *all* these *absurd* and *inaccurate* formulæ are still retained,—a *disgrace* to the editor, and an *insult*, as well as an *injury* to the public at large.

As the present *Appendix* may be considered as a continuation of my *Doctrine of Life Annuities and*

Assurances, the latter work will not, in future, be sold separately from the former; although the Appendix may still be had by the purchasers of the original treatise. The *whole work* is now become too bulky for *one* volume: and I have, therefore, for greater convenience, divided it into *two*, at the end of CHAPTER XI. By this arrangement all the *Analytical* part of the original treatise will be in the first volume: and the *Practical* part, together with the *Tables* and the present *Appendix*, will be in the second volume. New title-pages are printed corresponding to such division.

The latter part of the first note in page 458 having misled some persons, I have cancelled that page, together with the subsequent one, for the purpose of introducing others. The note, here alluded to, originally belonged to another method of solution; and was then correct. But, whilst the work was printing, I changed the mode of solution, and inadvertently retained the note in question, which does not apply thereto. The original method of solution is now restored; as, upon the whole, I think it preferable to the one which was afterwards substituted.

March 1, 1813. F. B.

Names of the Council of the Royal Society for the Year 1812.

Sir Joseph Banks, Bart. *Pres.*
Sir Charles Blagden.
Admiral William Bligh.
Bishop of Carlisle.
Anthony Carlisle, Esq.
Dr. (now Sir) Humphry Davy, *Sec.*
Lord Dundas.
Edward Howard, Esq.
Samuel Lysons, Esq. *Treas.*
Joseph de Mendoza Rios, Esq.
Matthew Montagu, Esq.
William Morgan, Esq.
Earl of Morton.
John Pond, Esq. *Astr. Roy.*
Lord Redesdale.
Edward Rudge, Esq.
Joseph Sabine, Esq.
Lord Webb Seymour.
Viscount Valentia.
Dr. Wm. Hyde Wollaston, *Sec.*
Dr. Thomas Young, *For. Sec.*

ON A NEW METHOD OF CALCULATING THE VALUE OF LIFE ANNUITIES.

Read June 4, 1812.

THE trouble attending the calculation of TABLES for determining the value of LIFE ANNUITIES, particularly where two or more lives are concerned, is so great that (notwithstanding the importance of the subject, and its extensive application and utility at the present day) few persons have been disposed to undertake the laborious task of forming them. If we except those tables which have been published by DR. PRICE and MR. BARON MASERES, there are no others that, in the present state of the science, can properly be applied to any practical use. The *former* has given, in his *Observations on Reversionary Payments*, several valuable tables deduced from the bills of mortality as observed at *Northampton* and in *Sweden:* and the *latter* has given, in his *Principles of the Doctrine of Life Annuities*, similar tables deduced from the observations of *M. De Parcieux* in France.

At the same time that DR. PRICE published the Tables above alluded to, he gave "A Specimen of an "easy and expeditious method of calculating the values "of Annuities on Single or Joint Lives;" which greatly

abridged the labour attending such calculations: and in my *Doctrine of Life Annuities and Assurances*, page 36, I have given an improvement of that method. The *practical* application, however, of this method is capable of being rendered much more simple and easy, as I shall attempt to show in the course of this paper. But the more immediate object that I have in view is to lay before the Society a *new* method of arranging the Tables for determining the value of Life Annuities: whereby a considerable portion of the time and labour employed in such calculations may be avoided, and their application to the solution of various problems, connected with the subject, rendered more extensive and easy.

Let A be the life upon which the annuity is granted; and let the number of persons living at the age of A, according to any given table of observations, and at 1, 2, 3, &c years older, be denoted respectively by a, b, c, d, ... w, x, y, z:—z denoting the number of persons living at the age of the oldest life in the given table, and in general equal to unity. Moreover let the *amount* of £1 in a year, according to the given rate of interest, be denoted by r. This being premised, it is well known (by those who are conversant with the subject) that the value of an annuity on the single life A is expressed by the following series:

$$\frac{1}{a}\left(\frac{b}{r}+\frac{c}{r^2}+\frac{d}{r^3}+\ldots\ldots\ldots\ldots\frac{x}{r^{n-2}}+\frac{y}{r^{n-1}}+\frac{z}{r^n}\right),$$

n denoting the number of years from the age of A to

the age of the oldest life in the table of observations: and the *sum* of this series, numerically expressed according to the age of A, forms the common tables of the value of Life Annuities. But the above series may be expressed, more conveniently for our present purpose, by the following one,

$$\frac{1}{ar^n}\left(br^{n-1}+cr^{n-2}+dr^{n-3}+\ldots\ldots xr^2+yr+z\right):$$

which is evidently the same as the former.

In the plan proposed, therefore, A is supposed to be a child just born; or one whose age is equal to 0: and *each* term of the series (beginning at the end) is to be numerically expressed and arranged in regular order in the same column; at the bottom of which must be placed the numerical value of the common divisor ar^n. In a collateral column are to be placed the *sums* of the first *one*, first *two*, first *three*, first *four*, &c values in the former column, which I shall denote by Z, Y, X, &c; and by the help of these two columns we shall be enabled to solve every question relative to Life Annuities and Assurances: not only with less liability to error, but frequently in a more expeditious and easy manner than by the usual method of proceeding.

In order to render these remarks the more intelligible, I shall insert the following Specimen, which shows the manner in which the values, above alluded to, are to be arranged; and where I have supposed the age of the oldest life in the table of observations to be 96 years.

Age.	Column. $\mathfrak{A}$	Column. $\mathfrak{B}$
96	z	Z
95	yr	Y
94	xr^2	X
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
3	dr^{n-3}	D
2	cr^{n-2}	C
1	br^{n-1}	B
0	ar^n	A

Consequently we shall have

$$Z = z$$
$$Y = yr + z$$
$$X = xr^2 + yr + z$$

.
.
.
.
.
.

$$C = cr^{n-2} + dr^{n-3} + er^{n-4} \ldots\ldots xr^2 + yr + z$$
$$B = br^{n-1} + cr^{n-2} + dr^{n-3} + \ldots\ldots xr^2 + yr + z$$
$$A = ar^n + br^{n-1} + cr^{n-2} + \ldots\ldots xr^2 + yr + z$$

Whence it follows that the value of an annuity on the life of a child *just born* is denoted by $\frac{B}{ar^n}$; that the value of an annuity on a life *one* year old is denoted by $\frac{C}{br^{n-1}}$; on a life *two* years old, by $\frac{D}{cr^{n-2}}$; on a life *three* years old, by $\frac{E}{dr^{n-3}}$; and so on to the extremity of human life.

It may be objected to this mode of arrangement that a *numerical operation* is necessary for determining the value of the annuity: whereas, in the usual method of arranging the tables, such value is found by *inspection**. This objection, however, is of weight only as far as regards *single* lives, and a few cases of *two joint* lives: because, in the cases of those joint lives whose ages are *not* given in the existing tables, the values are

* The mode of arranging Annuity Tables, as hitherto adopted by most *English* writers on this subject, (that is, by showing the *number of years purchase* that the annuities are worth) is certainly accompanied with many advantages, since it facilitates the solution of several problems that frequently arise. The *French* writers, however, give the *reciprocals* of those values; that is, they form their tables so as to show the *annuity which £1 will purchase.* But this is not so convenient for general use, as the former method: nevertheless it is frequently adopted, even in this country, and particularly by most of the public offices. The new method, here suggested, is in many cases superior to both; and may, in all cases, be readily converted into either, by changing the numerator for the denominator; and *vice versâ.* Thus, $\frac{B}{ar^n}$ is the value of an annuity on the life of A just born: and $\frac{ar^n}{B}$ is the annuity which £1 will purchase, on the same life.

not so readily found, by the method now in use, as by the process which is here laid down. Moreover, the present arrangement will enable us to solve, with great facility and expedition, a variety of intricate problems that frequently occur in practice, (particularly in cases of *deferred* or *temporary* annuities and assurances) which, according to the present method of solution, require much time and labour, and are consequently liable to considerable error in the numerical enunciation.

For instance, the value of an annuity on the life of a child just born, *deferred* for 1, 2, 3, &c years, is respectively denoted by $\frac{C}{ar^n}$, $\frac{D}{ar^n}$, $\frac{E}{ar^n}$, &c. The value of a similar annuity on a life *one* year old is respectively denoted by $\frac{D}{br^{n-1}}$, $\frac{E}{br^{n-1}}$, $\frac{F}{br^{n-1}}$, &c. On a life *two* years old, by $\frac{E}{cr^{n-2}}$, $\frac{F}{cr^{n-2}}$, $\frac{G}{cr^{n-2}}$, &c, and so on.

In like manner, the value of a *temporary* annuity, for 1, 2, 3, &c years on the life of a child just born is respectively denoted by $\frac{B-C}{ar^n}$, $\frac{B-D}{ar^n}$, $\frac{B-E}{ar^n}$, &c. The value of a similar annuity on a life *one* year old is respectively denoted by $\frac{C-D}{br^{n-1}}$, $\frac{C-E}{br^{n-1}}$, $\frac{C-F}{br^{n-1}}$, &c. On a life *two* years old, by $\frac{D-E}{cr^{n-2}}$, $\frac{D-F}{cr^{n-2}}$, $\frac{D-G}{cr^{n-2}}$, &c. and so on.

Moreover, if an annuity be *deferred* for any number of years and to be paid for in *equal annual payments**,

* The first payment to be made immediately, and the remaining ones at the beginning of each succeeding year.

the value of those annual payments may be determined in the same easy and expeditious manner. Thus, let the annuity be deferred for 5 years, on the life of a child just born, to be paid for in 5 equal annual payments, the value of those equal annual payments will be denoted by $\frac{G}{A-F}$: if the life had been *one* year old, the value would be denoted by $\frac{H}{B-G}$: and so on. Had the period been for 6 years, to be paid for in 6 equal annual payments, the values would be respectively $\frac{H}{A-G}$, and $\frac{I}{B-H}$: and so on, for any other interval.

The value of *Assurances* on any life may also be determined with nearly equal facility. Thus, the value of an assurance of £1 on the life of a child just born will be denoted by $\frac{A-B.r}{r.ar^n}$; on a life *one* year old, by $\frac{B-C.r}{r.br^{n-1}}$; on a life *two* years old, by $\frac{C-D.r}{r.cr^{n-2}}$: and so on.

If the assurance be paid for *in equal annual payments**, the value of those payments may be determined by substituting A, B, C, &c instead of ar^n, br^{n-1}, cr^{n-2}, &c, in the several denominators above stated. Thus, the value of an assurance of £1 on the life of a child just born, to be paid for *in one present payment*, is equal to $\frac{A-B.r}{r.ar^n}$: and the annual payment which ought to be made as an equivalent thereto during that life is $\frac{A-B.r}{A.r}=\frac{1}{r}-\frac{B}{A}$; which is a much more simple

* See the preceding note.

formula for such cases than any that has ever yet been given.

In a similar manner we may find the value of *deferred assurances.* Thus, the value of an assurance of £1 on the life of a child just born, *deferred* for 1, 2, 3, &c years, is respectively denoted by $\frac{B-C.r}{r.a^{rn}}$, $\frac{C-D.r}{r.ar^{n}}$, $\frac{D-E.r}{r.ar^{n}}$, &c. And on a life *one* year old, deferred for the same periods, it is respectively $\frac{C-D.r}{r.br^{n-1}}$, $\frac{D-E\,r}{r.br^{n-1}}$, $\frac{E-F.r}{r.br^{n-1}}$, &c. and so on.

A similar method of proceeding will lead us to the value of *temporary assurances.* Thus, the value of an assurance of £1 for 4 years, on the life of a child just born, is equal to $\frac{A-E-(B-F)r}{r.a^{rn}}$.

If this temporary assurance be paid for in *annual payments*, the value of those annual payments will be $\frac{1}{r}-\frac{B-F}{A-E}$: which is a much more simple and expeditious mode of obtaining the value than any hitherto known.

If what has been stated respecting *single* lives be well understood, no difficulty is likely to occur in the case of any number of *joint* lives. For, if we take *two* joint lives A, B, and make the number of persons living at the age of A, and at the age of 1, 2, 3, &c years older, equal to a, a', a'', a''', &c; and likewise make the number of persons living at the age of B, and at the age of 1, 2, 3, &c years older, equal to b, b', b'', b''', &c: then, by substituting ab for a, $a'b'$ for b, $a''\,b''$ for c, $a'''b'''$ for d, &c, in the above specimen,

we shall find that the same rule will enable us to determine with equal facility the value of an annuity on *two* joint lives of any ages. And the like method may be extended also to the case of *three* joint lives.

In order to find the *Expectation of Life* we have only to make $r=1$ in the specimen above given, and the same rule will still hold good; with this exception, however, that we must add $\frac{1}{2}$ to the values thus found, in order to obtain the true expectation of life for every *moment* of human existence, agreeably to the method laid down by all the writers on this subject. Thus, if we make

$$\begin{aligned} \text{A} &= a+b+c+ \;.\;.\;.\;.\;.\;.\; x+y+z \\ \text{B} &= b+c+d+ \;.\;.\;.\;.\;.\;.\; x+y+z \\ \text{C} &= c+d+e+ \;.\;.\;.\;.\;.\;.\; x+y+z \end{aligned}$$

then will the expectation of a *single* life A be equal to $\frac{\text{B}}{a}+\frac{1}{2}=\frac{\text{A}}{a}-\frac{1}{2}$: the expectation of a single life B will be equal to $\frac{\text{C}}{b}+\frac{1}{2}=\frac{\text{B}}{b}-\frac{1}{2}$: and so on, for any other life. And in like manner we might proceed to find the expectation of any *two joint* lives, after making the substitutions above alluded to. But it is presumed that enough has been said to show the application of the method.

Having thus given a general view of the principles upon which this new method is founded, I shall proceed to give some practical examples of its utility and superior advantage. For this purpose I have annexed to this paper four Tables which have been calculated

agreeably to the principles above alluded to, and which will more fully illustrate the rules above given. All these tables have been deduced from the *Sweden* observations of life *in general.*

The *first* table is for determining the *expectation* of a *single* life, at every age. The ages are inserted in the first column in an inverted order, beginning with the oldest life in the table of observations. In the column 𝔄, and opposite to the respective ages, are inserted the number of persons living at those ages; and in the column 𝔅 are set down the *sums* of the values in the preceding columns, according to the method above explained.

The *second* table is for determining the *expectation* of *two joint* lives whose difference of age is 9, 10, and 11 years; these three differences being requisite for the solution of some of the following questions. The age of the *youngest* of the two lives is inserted in the first column; and the values in the column 𝔄 are found by multiplying together the number of persons living at the respective ages of the two given lives. Thus, the number of persons living at the ages of 71 and 80 are 1622 and 558, the product of which is 905076; and which forms the seventeenth value in this column. The column 𝔅 is formed as in the preceding table by taking the *sums* of the values in column 𝔄. The column marked x in this and the subsequent tables will be explained hereafter.

The *third* table is for determining the value of an *an-*

nuity on a *single* life, at every age;—its construction will be sufficiently evident if compared with the specimen given in page 24.

The *fourth* table is for determining the value of an *annuity* on *two joint* lives, whose difference of age is 9, 10, and 11 years: it is formed upon similar principles to the preceding one. The age of the *youngest* of the two lives is inserted in the first column, as already explained in the second table.

The rate of interest made use of in the computation of the last two tables is 4 per cent.

These are all the tables which are necessary to be inserted for the purpose of solving the following cases; but I ought previously to explain the meaning of the column x in the last three tables. The figures in that column are of use in enabling us to *point off* the proper number of *integers* that arise from the process: they are the logarithmic indices of the numbers before which they stand, and consequently indicate the *places of figures* of which such numbers would consist if fully expressed. Thus, in the second table, the number against the age of 0, in column $\mathfrak{A}$, ought to be 61770000: but, as the first *six* figures are all that are necessary for our purpose, the last two figures are rejected, in order to abridge the size of the table. The logarithmic index 7, which is prefixed, shows therefore that the number ought to consist of *eight* places of figures, and that we must affix ciphers in order to make up that number. Where the index is *less* than

the number of figures, we have only to point off the proper number of integers, and to consider the rest as decimals.

This being premised, it is presumed that the principles upon which the following cases are solved, will be sufficiently clear and intelligible without any further explanation.

The *expectation* of a single life aged 40 is $\frac{110935}{4591}+\frac{1}{2}$ $=24{\cdot}664$; the *expectation* of a life of 50 is $\frac{69086}{3846}+\frac{1}{2}$ $=18{\cdot}463$: the *expectation* of two joint lives aged 40—50 is $\frac{246594800}{17657000}+\frac{1}{2}=14{\cdot}466$.

The value of an *annuity* on a *single* life aged 40 is $\frac{579207{\cdot}1}{41283{\cdot}3}=14{\cdot}030$; and the value of an *annuity* on a *single* life aged 50 is $\frac{272201{\cdot}4}{23363{\cdot}8}=11.651$. On the two *joint* lives aged 40—50, the value is $\frac{1034031000}{107263000}=$ 9.640.

The annuity *which* £1 *would purchase* on each of these lives would be respectively $\frac{41283{\cdot}3}{579207{\cdot}1}={\cdot}0713$, $\frac{23363{\cdot}8}{272201{\cdot}4}={\cdot}0858$, and $\frac{107263000}{1034031000}={\cdot}1037$.

The value of an annuity on a life of 40 *deferred* for 20 years is $\frac{105513{\cdot}5}{41283{\cdot}3}=2{\cdot}556$; and on the life of 50 *deferred* for 30 years it is $\frac{3411{\cdot}076}{23363{\cdot}8}={\cdot}146$. On the two joint lives 40—50 *deferred* for 15 years the value is $\frac{158765800}{107263000}=1{\cdot}480$.

The value of an annuity on a life of 40, de-

ferred for 20 years, and to be paid for by *equal annual payments* during that interval, is $\frac{105513\cdot5}{620490\cdot4-117538\cdot0}=$ ·2098; and the value of a similar annuity on a life of 50 deferred for 30 years, is $\frac{3411\cdot076}{295565\cdot2-4456\cdot200}=\cdot0117$. On the two joint lives 40—50, deferred for 15 years and payable in a similar manner, the value is $\frac{158765800}{1141294000-185786900}=\cdot1661$.

The value of a *temporary* annuity for 20 years on a life aged 40 is $\frac{579207\cdot1-105513\cdot5}{41283\cdot3}=11\cdot474$: and of a *temporary* annuity for 30 years on a life aged 50 is $\frac{272201\cdot4-3411\cdot076}{23363\cdot8}=11\cdot505$. On the two joint lives 40—50, for 15 years, the value is $\frac{1034031000-158765800}{107263000}$ $=8\cdot160$.

The value of an *assurance* of £1 on a life of 40 is $\frac{620490\cdot4-579207\cdot1\times1\cdot04}{41283\cdot3\times1\cdot04}=\cdot4219$: and the value of a similar *assurance* on a life of 50 is $\frac{295565\cdot2-272201\cdot4\times1\cdot04}{23363\cdot8\times1\cdot04}=\cdot5134$. On the two joint lives 40—50, it is $\frac{1141294000-1034031000\times1\cdot04}{107263000\times1\cdot04}=$ ·5908.

The value of an *assurance* of £1 in *annual payments* on a single life, aged 40, is equal to $\cdot96154-\frac{579207\cdot1}{620490\cdot4}=\cdot02807$: the value of a similar assurance on a life, aged 50, is equal to $\cdot96154-\frac{272201\cdot4}{295565\cdot2}=\cdot04059$. On two joint lives 40—50, it is equal to $\cdot96154-\frac{1034031000}{1141294000}=\cdot05552$.

The value of an *assurance* of £1 on a life of

40, *deferred* for 20 years, is $\frac{117538\cdot0-105513\cdot5\times1\cdot04}{41283\cdot3\times1\cdot04}$ $=\cdot1818$: and on a life of 50, *deferred* for 30 years, it is $\frac{4456\cdot200-3411\cdot076\times1\cdot04}{23363\cdot8\times1\cdot04}=\cdot0374$. On the two joint lives 40—50, *deferred* for 15 years, the value is $\frac{185786900-158765800\times1\cdot04}{107263000\times1\cdot04}=\cdot1853$.

The value of a *temporary assurance* of £1 on a single life, aged 40, for 20 years, is equal to $\frac{620490\cdot4-117538\cdot0-1\cdot04(579207\cdot1-105513\cdot5)}{41283\cdot3\times1\cdot04}=\cdot2401$: and on a life aged 50, for 30 years it is equal to $\frac{295565\cdot2-4456\cdot200-1\cdot04(272201\cdot4-3411\cdot076)}{23363\cdot8\times1\cdot04}=\cdot4760$. On two joint lives 40—50 for 15 years it is equal to $\frac{1141294000-185786900-1\cdot04(1034031000-158765800)}{107263000\times1\cdot04}=$ $\cdot4055$.

If we apply these tables to the solution of any cases in *Contingent* annuities and assurances, the advantage of their form and arrangement will be much more evident; since we can, by a very simple numerical operation, solve some of the most intricate cases on this subject.—In questions relative to contingent annuities it is always required to know the *probability of survivorship* between the given lives; but according to the methods hitherto adopted, this information cannot be obtained except at an expense of time and labour which few persons are disposed to give to the subject. Neither are there any tables in existence that will suit all the cases that may occur. By the present plan, however, this may be very readily obtained.

Let us take the case of two lives A, B; of which A

is supposed to be the youngest. And let us suppose A′ to be a life one year older than A, and B′ to be a life one year older than B. Then if we make AB, A′B, AB′, A′B′, to denote the values in Table II standing against the ages of the joint lives AB, A′B, AB′, A′B′ respectively in the column $\mathfrak{B}$: and if we make *ab* to denote the value standing against the ages of the joint lives AB in the column $\mathfrak{A}$, we shall have the following formula:

$$\frac{AB - A'B - (A'B' - AB')}{2ab}$$

for the value of the probability that the life A shall die before the life B. So that, if the life of A were 40, and of B 50, the value would be equal to

$$\frac{264251800 - 257791100 - (246594800 - 252670600)}{2 \times 17657000} = \frac{12536500}{35314000}$$

$= \cdot 3550.$

If we retain the same characters to denote the similar and corresponding values in Table IV, we shall have the following formula:

$$\frac{AB - A'B - r(A'B' - AB')}{2r.abr^n}$$

for the value of an assurance of £1 on the life of A provided he dies *before* B: the numerical solution of which formula is readily performed by the help of the present tables.—For instance, let us suppose the age of A to be 40, and of B 50: and that it were required to determine the value of an assurance of £1 on the life of 40, provided he dies *before* the life of 50. This value would be equal to

$$\frac{1141294000 - 1116533000 - 1 \cdot 04(1034031000 - 1056721000)}{2 \cdot 08 \times 107263000}$$

$= \frac{48358600}{223107040} = \cdot 2168.$

If the value of the assurance be required in *annual payments*, we have only to substitute AB instead of abr^n in the denominator of the given formula. Thus, $\frac{48358600}{223107040}$ being the value of the assurance in one present payment, the value of the same in equal annual payments will be $\frac{48358600}{2373891520} = \cdot 02037$.

In like manner we might proceed to find the value of an assurance depending on any order of survivorship amongst *three* lives* : for, if a Table were formed for *three joint* lives upon similar principles to those which have been already explained, and if we retained the same characters as before to denote the similar and corresponding values in that Table, the formulæ for such cases might be rendered equally simple and easy. Thus, the present value of £1 payable on the decease of A, provided he be the *first* that fails of three given lives, A, B, C, would be denoted by

$$\frac{2ABC - 2A'BC + AB'C - A'B'C - r(2A'B'C' - 2AB'C' + A'BC' - ABC')}{6r.abcr^n}$$

which produces the same value as the formula given in page 211 of my *Doctrine of Life Annuities*, &c. And the present value of £1, payable on the decease of A or B provided *either* of them be the *first* that fails of three given lives A, B, C, would be denoted by

$$\frac{4ABC - A'BC - AB'C - 2A'B'C - r(4A'B'C' - AB'C' - A'BC' - 2ABC',)}{6r.abcr^n}$$

which produces the same value as the formula given in

* This part relative to *three* lives was not inserted in the paper when it was sent to the Royal Society.

page 240 of my work above mentioned. These two formulæ, together with those already given, will serve for the solution of *most* cases in Contingent Assurances where three joint lives are concerned* : and in those cases where we are obliged to resort to other methods, the process will be found to be equally simple and easy.

If the assurance be required for a *term of years* only, the value may *in all cases* be determined with *equal* ease and expedition.

Let any person attempt to solve the cases above given, by the common rules laid down for that purpose, and by any of the existing tables upon this subject, and he will immediately be convinced of the superior advantage of the plan here proposed. Almost all of them, particularly where two or more joint lives are concerned, would require the aid of logarithms and a troublesome calculation; and the solution would consequently be exposed to considerable error; whereas, agreeably to the present plan, the greater part of the process is performed simply by subtraction, and *only one* operation by *division* is required.

If we add another column $\mathfrak{C}$, to the one already given in the specimen in page 24, and take $\mathfrak{z}$, $\mathfrak{y}$, $\mathfrak{x}$, &c to denote the *sums* of the values in column $\mathfrak{B}$; then by arranging them in a similar manner, as in the following specimen,

* They will serve for the solution of Problems 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 43 and 46, in my *Doctrine of Life-Annuities, &c.*

Age.	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$
96	z	Z	$\mathfrak{z}$
95	yr	Y	$\mathfrak{y}$
94	xr^2	X	$\mathfrak{x}$
.			
.			
.			
.			
.			
3	dr^{n-3}	D	$\mathfrak{d}$
2	cr^{n-2}	C	$\mathfrak{c}$
1	br^{n-1}	B	$\mathfrak{b}$
0	ar^n	A	$\mathfrak{a}$

we shall consequently have

$\mathfrak{z}=$Z

$\mathfrak{y}=$Y$+$Z

$\mathfrak{x}=$X$+$Y$+$Z

&c.

In this case, $\frac{\mathfrak{b}}{ar^n}$ will be the value of an *increasing* annuity on the life of a child *just born:* that is, of an annuity of *one* pound for the first year, *two* pounds for the second year, *three* pounds for the third year; and so on. Moreover, $\frac{\mathfrak{c}}{br^{n-1}}$ will be the value of a similar annuity on the life of a child *one* year old:

$\frac{v}{cr^{n-2}}$ will be the value of a similar annuity on the life of a child *two* years old: and so on, for any other age.

These are some of the principal advantages attending the new arrangement of the tables. There are others, of inferior moment, which it would be needless to insist upon in this place, since they will immediately occur to such persons as have turned their attention to this subject. Enough has probably been said to warrant the adoption of this method in the formation of any *new* tables which may be hereafter published: and the following directions may be of use to those persons who are employed in such calculations.

Provide a set of smooth deal lathes or rods, one edge of which should be about one fifth of an inch in thickness; and divide the whole length of each rod into a convenient number of equal parts. On the edge of the rod, thus divided, (or on a slip of paper, to be pasted thereon) write down, within the several divisions, the logarithms of the number of persons living at every age of human existence: but, in an *inverse* order; that is, beginning at the age of the *oldest* life in the table of observations, and proceeding upwards to the *youngest*. If it be required to calculate the value of an annuity on *two joint* lives, *two* similar sets of such rods must be formed: and if *three joint* lives, then *three* similar sets; and so on. Let the computist a so prepare another set of rods, divided in a similar manner, whereon he must insert the logarithms of the

amount of £1 at the end of 0, 1, 2, 3, &c years; according to the given rate of interest. Thus, let it be required to calculate the value of an annuity on two joint lives, according to the *Northampton* table of observations, and at 4 per cent interest; in which case the logarithms on the respective rods will be arranged in the following manner:

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c

·0000000	·0170333	·0340667	·0511000	·0681334	·0851667	&c

And when we operate with these rods, they must be brought together (either by the hand, or by means of a groove in the table) for the more convenient addition of the numbers in the respective columns. If the two lives are of equal ages, the ends of all the rods must coincide with each other, in which case the logarithms in the several divisions will stand directly under each other; as in the following manner:

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c
·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c
·0000000	·0170333	·0340667	·0511000	·0681334	·0851667	&c

and the natural numbers, corresponding to the sums of the logarithms in each perpendicular division, will be the values that are to be inserted in the column A in the Table.

If there be *one* year's difference between the ages of

the two lives, the upper rod must be moved *one* division towards the left, as in the following manner:

·0000000	·6020600	·9542425	1·2041200	1·3802112	1·5314789	&c
	·0000000	·6020600	·9542425	1·2041200	1·3802112	&c
	·0000000	·0170333	·0340667	·0511000	·0681334	&c

and the natural numbers, corresponding to the sums of the logarithms in each perpendicular division, will be the values that are, in this case, to be inserted in the column $\mathfrak{A}$ in the Table.

If there be *two* years difference between the ages of the two lives, the upper rod must be moved *two* divisions towards the left; if *three* years, *three* divisions; and so on: and the natural numbers corresponding to the sums in these cases, will be the values that ought to be inserted in the column $\mathfrak{A}$. So that these three sets of rods will serve for *every possible combination* of ages between two lives: and thereby not only save an immense deal of trouble, but moreover prevent those errors which are so likely to occur when we have to transcribe the same numbers over and over again with the pen. It may be further remarked that the rod which contains the logarithms of the amount of £1, may be used with *any other* table of observations, where the same rate is required. In the same way that the rods, containing the logarithms of the number of persons living, may be used with any other *rate* that is required.

In order to detect, by a different process, any error in taking out and operating by the logarithms, we may

adopt the following method*. Let Z, Y, X, &c respectively denote the *logarithm* of the number of people living, in the table of observations, at the age of the oldest life, and at 1, 2, 3, &c years younger than the oldest: and let R denote the *logarithm* of the amount of £1 in a year, according to the given rate of interest. Then the logarithms, for finding the value of an annuity on any two *equal* joint lives, will be

$Z+Z$, $Y+Y+R$, $X+X+2R$, $W+W+3R$, $V+V+4R$, &c.

Having thus found the logarithms for such cases where the ages are equal, or where the difference of age is = 0, we may find the logarithms for those cases where the difference is 1, 2, 3, 4, &c years, by the following method. Take out the *differences* between the logarithms Z, Y, X, &c, thus,

$$Y-Z,\ X-Y,\ W-X,\ V-W,\ T-V,\ \&c$$

and *add* them respectively to the values above found, in the manner stated below; and the results will give the proper logarithms for finding the value of an annuity on any two joint lives whose difference of age is respectively 1, 2, 3, &c years. Thus, the logarithms, when the difference of age is

0 year are =	$Z+Z$,	$Y+Y+R$,	$X+X+2R$,	$W+W+3R$, &c
add	$Y-Z$	$X-Y$	$W-X$	$V-W$
1 year are =	$Z+Y$,	$Y+X+R$,	$X+W+2R$,	$W+V+3R$, &c
add	$X-Y$	$W-X$	$V-W$	$T-V$
2 years are =	$Z+X$,	$Y+W+R$,	$X+V+2R$,	$W+T+3R$, &c
add	$W-X$	$V-W$	$T-V$	$S-T$
3 years are =	$Z+W$,	$Y+V+R$,	$X+T+2R$,	$W+S+3R$, &c

* This method of proving the accuracy of the calculation has

It will be readily seen that the results, obtained by this process, will be the same as by the rods above alluded to, provided they are both accurate. But if the results should differ, there must be some error, which ought to be corrected.

The public are indebted to Mr. George Barrett, of Petworth in Sussex, for this easy, expeditious and ingenious mode of determining and arranging the value of Life Annuities: and it is with his concurrence that I have drawn up this paper to explain the principles upon which it is founded and to point out the advantages which attend it. I have been the more induced to enter upon this subject, since it gives me an opportunity of perpetuating the name of one who, by his labours on this subject, has deserved so much from every person that is interested in the science: and at the same time of making known to the world the astonishing and beneficial effects that may be produced by perseverance and attention in so laudable a pursuit. Amidst a variety of avocations Mr. Barrett has never lost sight of his favourite object. At an early period of his life he became attached to this branch of analysis; and without the aid of a master, he made considerable progress in the mathematics, with a view to his further and more complete study of the subject. He very soon saw, and lamented how much the application of the science was abridged, through the want of a more nu-

been communicated to the author, *since* the Paper was read before the Royal Society. See the note in page 48.

merous and extensive set of Life Annuity Tables than those we at present possess. In order to remedy this defect he at first devoted his leisure hours to the formation of a few tables which were intended for a new society that was about to be established in his neighbourhood. Pleased with the subject, and probably intending to render his labours more serviceable to the public at large, he extended his plan; and after twenty-five years close application he has calculated the most complete set of tables relative to this subject that probably will ever be again produced by any one person. These Tables are eighteen in number, and may be arranged in the following manner:

For the *Expectation* of Lives.

TABLES I, II, III, IV, V. Showing the number of persons living at the several ages therein mentioned; according to the observations made in *Sweden* amongst *males*, amongst *females*, and amongst *lives in general*; according to the observations made at *Northampton*; and according to the observations made by *M. De Parcieux* in France. From which may be easily deduced the Expectation of life at the several ages, in each place respectively.

TABLE VI. For ascertaining the Expectation of any *two joint* lives, and also the probability of one life dying *before* or *after* another: deduced from the *Sweden* table of lives *in general*.

TABLE VII. For ascertaining the Expectation of a

Single life, or of any *Two*, *Three*, or *Four* Joint lives, deduced from *M. De Moivre's* Hypothesis of an equal decrement of life.

For Annuities on *Single* lives.

TABLE VIII. For ascertaining the value of an annuity on the life of a *Male* at every age; interest at 3, 4, 5 and 6 per cent, deduced from the *Sweden* table of observations.

TABLE IX. For ascertaining the value of an annuity on the life of a *Female* at every age; interest at 3, 4, 5 and 6 per cent, deduced from the *Sweden* table of observations.

TABLE X. For ascertaining the value of an annuity on a Single life, at every age; interest at 3, 3½, 4, 4½, 5, 5½, 6, 6½, 7, 7½ and 8 per cent, deduced from the *Sweden* table of lives *in general.*

TABLE XI. For ascertaining the value of an annuity on a Single life, at every age; interest at 3, 3½, 4, 4½, 5, 5½, and 6 per cent, deduced from the *Northampton* table of observations.

TABLE XII. For ascertaining the value of an annuity on a Single life, at every age; interest 3, 3½, 4, 4½, 5, 5½, and 6 per cent, deduced from the observations of *M. De Parcieux* in France.

For Annuities on *Two Joint* lives.

TABLE XIII. For ascertaining the value of an annuity on Two joint lives, consisting of a *Male* and a

Female, for every possible combination of age; interest at 4 per cent, deduced from the *Sweden* table of observations.

TABLE XIV. For ascertaining the value of an annuity on Two joint lives, for every possible combination of age; interest at 3, 4, 5 and 6 per cent, deduced from the *Sweden* table of lives *in general.*

TABLE XV. For ascertaining the value of an annuity on Two Joint lives, for every possible combination of age; interest at 3, 4 and 5 per cent, deduced from the *Northampton* table of observations.

TABLE XVI. For ascertaining the value of an annuity on Two Joint lives, for every possible combination of age; interest at 3, 4 and 5 per cent, deduced from the observations of *M. De Parcieux.*

For Annuities on *Three Joint* lives.

TABLE XVII. For ascertaining the value of an annuity on Three Joint lives, for every possible combination of age; interest at 4 per cent, deduced from the *Sweden* table of lives *in general.*

For Annuities on *M. De Moivre's* Hypothesis.

TABLE XVIII. For ascertaining the value of an annuity on a *Single* life, and on *Two*, *Three*, and *Four* joint lives, for every possible combination of age; interest at 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, $7\frac{1}{2}$ and 8 per cent, deduced from *M. De Moivre's* Hypothesis of an equal decrement of life.

All the above Tables are arranged agreeably to the principles laid down in this paper; and will therefore show, with *equal* facility, the value of annuities, whether they be *temporary*, *deferred*, or for the *whole duration* of life. Tables VIII, IX, and XIII, have the additional column $\mathfrak{C}$, alluded to in page 38, whereby they show the value of *increasing* annuities; and are consequently of essential use to those Societies which are formed for the benefit of *Old Age* and of *Widows*. Tables XI and XV will be extremely useful to the various Public Companies lately established for the purpose of granting *Assurances* on lives: and will save much time and labour in estimating, at the stated intervals, the true assets of such companies, agreeably to the method which I have laid down upon this subject in my *Doctrine of Life Annuities and Assurances*, page 456. Tables XII and XVI will be equally desirable to those societies which grant *Annuities* on lives: since they are formed from those observations of human mortality which show more correctly than any other the probability of living amongst a set of *Life Annuitants*. Tables X, XIV and XVII will be of very considerable use and advantage to the various *Colleges* and *Corporate Bodies* in this kingdom, as well as to noblemen and other persons of landed property, whose estates are *leased on lives*: since there are no tables at present existing, whereby the true value for the renewal of a life in such leases, can be fairly estimated; and we are consequently oftentimes obliged to rest contented

with an approximation which (there is much reason to fear) is frequently far from the truth. By the help of these tables, however, this doubt will be removed: and the values may in all cases be determined with the greatest accuracy. The remaining tables, which far exceed in extent any thing of the kind as yet published, will tend to the solution of many problems which have hitherto remained inexplicable for want of such assistance. The whole of them have been carefully examined and corrected separately by two other persons, so that there is the greatest reason to believe that they are accurately and truly computed*.

It is needless for me to enlarge, in this place, upon the astonishing perseverance and industry displayed by Mr. Barrett in the formation of these tables. Every one conversant with the subject must be aware of the immense time and trouble which must have been employed in the computation. It is indeed the most successful effort, of this kind, that has ever yet been made by any one individual: and every person interested in the science, must regret not only that they have not yet been made public, but that there is a chance of their being lost *for ever* to the world. Whether Mr. Barrett however may meet with sufficient patronage to induce him to publish the *whole* of these tables, or whether

* One of these persons operated by means of the rods alluded to in page 39; and the other by the method explained in page 42: the similarity of the two results confirmed the correctness of each other.

he may publish them *in part*, or whether indeed he may be prevailed upon to arrange them according to the usual method (for they are at all times capable of being thus converted)—the object and design of the present paper will be the same: which is, to point out some curious properties and advantages which such a mode would obtain over the present arrangement; and thus, by encouraging their adoption, to open a new field to the analyst. On the other hand, should Mr. Barrett fail in his object, it may not be considered altogether useless to hold out a *beacon* to posterity, lest they should pursue a similar hopeless course, and waste the best part of their lives in the same fruitless and delusive career.

TABLES.

TABLE I.

For the *Expectation* of a Single Life.

Age.	Column A.	Column B.	Age.	Column A.	Column B.
96	1	1			
95	2	3	65	2354	24961
94	5	8	64	2472	27433
93	11	19	63	2590	30023
92	21	40	62	2708	32731
91	33	73	61	2822	35553
90	47	120	60	2930	38483
89	62	182	59	3030	41513
88	82	264	58	3125	44638
87	109	373	57	3220	47858
86	144	517	56	3312	51170
85	189	706	55	3403	54573
84	244	950	54	3494	58067
83	309	1259	53	3584	61651
82	384	1643	52	3674	65325
81	468	2111	51	3761	69086
80	558	2669	50	3846	72932
79	648	3317	49	3924	76856
78	743	4060	48	3997	80853
77	848	4908	47	4069	84922
76	963	5871	46	4143	89065
75	1084	6955	45	4219	93284
74	1214	8169	44	4297	97581
73	1354	9523	43	4375	101956
72	1489	11012	42	4453	106409
71	1622	12634	41	4526	110935
70	1749	14383	40	4591	115526
69	1873	16256	39	4651	120177
68	1997	18253	38	4709	124886
67	2118	20371	37	4767	129653
66	2236	22607	36	4825	134478

Age.	Column A.	Column B.	Age.	Column A.	Column B.
35	4884	139362	15	5899	248597
34	4947	144309	14	5936	254533
33	5010	149319	13	5974	260507
32	5072	154391	12	6016	266523
31	5132	159523	11	6061	272584
30	5191	164714	10	6115	278699
29	5246	169960	9	6177	284876
28	5301	175261	8	6250	291126
27	5355	180616	7	6337	297463
26	5407	186023	6	6442	303905
25	5457	191480	5	6564	310469
24	5507	196987	4	6707	317176
23	5555	202542	3	6952	324128
22	5603	208145	2	7296	331424
21	5650	213795	1	7805	339229
20	5697	219492	0	10000	349229
19	5740	225232			
18	5782	231014			
17	5822	236836			
16	5862	242698			

TABLE II.

For the *Expectation* of Two Joint Lives.

Part 1.—*Difference of Age 9 Years.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
87	2	109000	2	1090000					
86		288000		3970000					
85		945000	3	1342000	55	6	841222	7	7291695
84	3	268400		4026000	54		904946		8196641
83		648900	4	1051500	53		970547		9167189
82	4	126720		2318700	52	7	103680	8	1020399
81		219960		4518300	51		110197		1130596
80		345960		7977900	50		116534		1247130
79		531360	5	1329150	49		122625		1369755
78		809870		2139020	48		128703		1498459
77	5	122112		3360140	47		134765		1633224
76		182007		5180210	46		140986		1774210
75		264496		7825170	45		147412		1921622
74		375126	6	1157643	44		154004		2075627
73		519936		1677579	43		160737		2236364
72		696852		2374431	42		167477		2403841
71		905076		3279507	41		174070		2577911
70	6	113335		4412860	40		180151		2758062
69		139164		5804499	39		185900		2943963
68		169346		7497955	38		191609		3135572
67		203963		9537589	37		197497		3333069
66		242382	7	1196141	36		203567		3536635
65		285776		1481917	35		209865		3746501
64		334709		1816626	34		216431		3962932
63		385651		2202277	33		223095		4186027
62		439238		2641514	32		229559		4415586
61		493568		3135082	31		235610		4651196
60		548789		3683871	30		241433		4892630
59		605091		4288962	29		247034		5139664
58		661875		4950837	28		252699		5392362
57		719992		5670829	27		258379		5650741
56		779645		6450474	26		264078		5914819

TABLE II. *continued.*

Yo^st. Age.	x.	Column 𝔄.	x.	Column 𝔅.	Yo^st. Age.	x.	Column 𝔄.	x.	Column 𝔅.
25	7	269958	8	6184777	10	7	351001	9	1089624
24		275901		6460677	9		357154		1125340
23		281750		6742427	8		363875		1161727
22		287546		7029973	7		371475		1198875
21		293291		7323264	6		380014		1236876
20		298865		7622129	5		389639		1275840
19		304277		7926406	4		400676		1315908
18		309526		8236033	3		418232		1357731
17		314796		8550828	2		442210		1401952
16		319889		8870717	1		477276		1449679
15		324858		9195575	0		617700		1511449
14		329745		9525320					
13		334723		9860043					
12		339904	9	1019995					
11		345295		1054524					

TABLE II. *continued.*

For the *Expectation* of Two Joint Lives.

Part 2.—*Difference of Age* 10 *Years.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
86	2	144000	2	1440000					
85		378000		5220000	55	6	801066	7	6720592
84	3	122000	3	1742000	54		863717		7584309
83		339900		5141000	53		928256		8512565
82		806400	4	1320500	52		994919		9507484
81	4	154440		2864900	51	7	106135	8	1056884
80		262260		5487500	50		112688		1169572
79		401760		9505100	49		118897		1288469
78		609260	5	1559770	48		124906		1413375
77		924320		2484090	47		131022		1544397
76	5	138672		3870810	46		137216		1681613
75		204876		5919570	45		143573		1825186
74		296216		8881730	44		150137		1975323
73		418386	6	1306559	43		156800		2132122
72		571776		1878335	42		163603		2295726
71		759096		2637431	41		170223		2465948
70		975942		3613373	40		176570		2642518
69	6	121370		4827077	39		182505		2825024
68		148377		6310848	38		188219		3013242
67		179606		8106912	37		193969		3207211
66		215327	7	1026018	36		199900		3407111
65		255174		1281192	35		206056		3613167
64		300101		1581292	34		212573		3825740
63		350686		1931978	33		219187		4044927
62		403221		2335200	32		225856		4270783
61		457728		2792928	31		232274		4503058
60		512457		3305385	30		238319		4741377
59		567519		3872904	29		243991		4985368
58		624062		4496966	28		249624		5234992
57		681996		5178962	27		255273		5490265
56		740563		5919526	26		260888		5751153

TABLE II. *continued.*

Yo^st. Age.	x.	Column 𝔄.	x.	Column 𝔅.	Yo^st. Age.	x.	Column 𝔄.	x.	Column 𝔅.
25	7	266520	8	6017673	10	7	348372	9	1068319
24		272431		6290104	9		354560		1103775
23		278305		6568409	8		361375		1139912
22		284184		6852594	7		368940		1176806
21		289958		7142552	6		377630		1214569
20		295731		7438283	5		387210		1253290
19		301120		7739403	4		398127		1293103
18		306504		8045907	3		415312		1334634
17		311768		8357675	2		438927		1378527
16		316958		8674634	1		473061		1425833
15		321908		8996542	0		611500		1486983
14		326895		9323437					
13		331856		9655293					
12		337076		9992370					
11		342446	9	1033482					

TABLE II. *continued.*

For the *Expectation* of Two Joint Lives.

Part 3.—*Difference of Age* 11 *Years.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
85	2	189000	2	1890000	55	6	760911	7	6163082
84		488000		6770000	54		822488		6985570
83	3	154500	3	2222000	53		885965		7871535
82		422400		6446000	52		951566		8823101
81		982800	4	1627400	51	7	101848		9841580
80	4	184140		3468800	50		108534	8	1092692
79		304560		6514400	49		114973		1207665
78		460660	5	1112100	48		121109		1328774
77		695360		1807460	47		127156		1455931
76	5	104967		2857130	46		133405		1589335
75		156096		4418090	45		139733		1729068
74		229446		6712550	44		146227		1875295
73		330376	6	1001631	43		152862		2028158
72		460101		1461732	42		159595		2187753
71		622848		2084580	41		166285		2354039
70		818532		2903112	40		172667		2526706
69	6	104513		3948246	39		178877		2705584
68		129406		5242302	38		184781		2890365
67		157367		6815976	37		190537		3080902
66		189613		8712104	36		196329		3277231
65		226690	7	1097901	35		202344		3479575
64		267965		1365865	34		208714		3688289
63		314426		1680291	33		215280		3903569
62		366663		2046955	32		221900		4125469
61		420196		2467150	31		228528		4353997
60		475246		2942396	30		234945		4588941
59		529947		3472343	29		240844		4829785
58		585312		4057656	28		246549		5076335
57		643034		4700690	27		252167		5328502
56		701482		5402171	26		257752		5586253

TABLE II. *continued.*

Yo^st. Age.	r.	Column A.	r.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
25	7	263300	8	5849554	10	7	345497	9	1046795
24		268962		6118515	9		351904		1081985
23		274806		6393321	8		358750		1117860
22		280710		6674032	7		366405		1154501
21		286568		6960600	6		375053		1192006
20		292370		7252970	5		384782		1230484
19		297963		7550933	4		395646		1270049
18		303324		7854257	3		412671		1311316
17		308624		8162881	2		435863		1354902
16		313910		8476791	1		469549		1401857
15		318959		8795750	0		406100		1462467
14		323927		9119677					
13		328988		9448666					
12		334189		9782854					
11		339598	9	1012245					

TABLE III.

For *Annuities* on a Single Life.

Age.	x.	Column A.	x.	Column B.	Age.	x.	Column A.	x.	Column B.
96	0	100000	0	1000000					
95		208000		3080000	65	3	794036	4	6598143
94		540800		8488000	64		867192		7465335
93	1	123735	1	2086150	63		944931		8710265
92		245670		4542853	62	4	102750		9437766
91		401496		8557808	61		111359	5	1055135
90		594700	2	1450481	60		120245		1175380
89		815878		2266358	59		129323		1304703
88	2	112223		3388585	58		138713		1443416
87		155141		4939995	57		148647		1592063
86		213155		7071547	56		159010		1751073
85		290957		9981115	55		169914		1920987
84		390652	3	1388763	54		181436		2102423
83		514508		1903271	53		193554		2295977
82		664964		2568235	52		206351		2502327
81		842842		3411076	51		219687		2722014
80	3	104512		4456200	50		233638		2955652
79		126224		5718440	49		247911		3203563
78		150518		7223622	48		262624		3466187
77		178661		9010230	47		278049		3744236
76		211005	4	1112028	46		294430		4038666
75		247019		1359047	45		311824		4350490
74		287708		1646755	44		330293		4680783
73		333723		1980477	43		349740		5030522
72		381676		2362153	42		370214		5400737
71		432399		2794552	41		391335		5792071
70		484905		3279457	40		412833		6204904
69		540055		3819512	39		434957		6639862
68		598841		4418353	38		457997		709[illegible]858
67		660530		5078883	37		482183		7580042
66		725224		5804107	36		507572		8087614

TABLE III. *continued.*

Age.	x.	Column A.	x.	Column B.	Age.	x.	Column A.	x.	Column B.
35	4	534330	5	8621943	15	5	141410	6	2734458
34		562871		9184814	14		147988		2882447
33		592841		9777655	13		154893		3037340
32		624185	6	1040184	12		162221		3199561
31		656831		1105867	11		169972		3369534
30		690958		1174963	10		178346		3547880
29		726210		1247584	9		187361		3735240
28		763176		1323901	8		197158		3932398
27		801789		1404080	7		207898		4140296
26		841957		1488276	6		219797		4360093
25		883733		1576649	5		232918		4593011
24		927503		1669400	4		247512		4840522
23		973011		1766701	3		266815		5107337
22	5	102068		1868768	2		291218		5398556
21		107041		1975809	1		323996		5722552
20		112249		2088058	0		431718		6154270
19		117619		2205677					
18		123219		2328896					
17		129034		2457931					
16		135118		2593049					

TABLE IV.

For *Annuities* on Two Joint Lives.

Part 1.—*Difference of Age* 9 *Years.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
87	2	109000	2	1090000					
86		299520		4085200					
85	3	102211	3	1430632	55	7	295105	8	2083078
84		301913		4449767	54		330159		2413237
83		759121	4	1204098	53		368256		2781493
82	4	154174		2745841	52		409132		3190625
81		278320		5529037	51		452242		3642867
80		455260	5	1008163	50		497377		4140244
79		727203		1735366	49		544309		4684554
78	5	115270		2888064	48		594142		5278696
77		180756		4695620	47		647011		5925706
76		280191		7497534	46		703953		6629660
75		423467	6	1173220	45		765478		7395138
74		624612		1797832	44		831700		8226838
73		900361		2698193	43		902785		9129623
72	6	125499		3953184	42		978264	9	1010789
71		169519		5648375	41	8	105744		1116533
70		220766		7856032	40		113816		1230349
69		281920	7	1067524	39		122146		1352495
68		356786		1424309	38		130933		1483429
67		446909		1871218	37		140355		1623783
66		552333		2423551	36		150455		1774239
65		677265		3100816	35		161315		1935554
64		824962		3925778	34		173016		2108570
63		988541		4914319	33		185477		2294047
62	7	117094		6085255	32		198485		2492532
61		136840		7453657	31		211866		2704398
60		158236		9036018	30		225786		2930184
59		181449	8	1085051	29		240265		3170449
58		206416		1291466	28		255605		3426055
57		233522		1524988	27		271805		3697860
56		262985		1787973	26		288912		3986772

TABLE IV. *continued.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
25	8	307159	9	4293930	10	8	719243	10	1182717
24		326477		4620408	9		761125		1258829
23		346734		4967142	8		806466		1339476
22		368022		5335165	7		856242		1425100
21		390391		5725556	6		910961		1516196
20		413722		6139278	5		971396		1613336
19		438063		6577341	4	9	103887		1717223
18		463594		7040935	3		112776		1829999
17		490188		7531123	2		124012		1954011
16		518044		8049167	1		139199		2093210
15		547134		8596301	0		187360		2280571
14		577579		9173881					
13		609752		9783632					
12		643957	10	1042759					
11		680337		1110793					

TABLE IV. *continued.*

For *Annuities* on Two Joint Lives.

Part 2.—*Difference of Age* 10 *Years.*

Yo^st. Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
86	2	144000	2	1440000					
85		393120		5371200	55	7	270210	8	1857869
84	3	131955	3	1856672	54		302997		2160865
83		382341		5680085	53		338663		2499529
82		943374	4	1511382	52		377504		2877032
81	4	187900		3390381	51		418820		3295852
80		331843		6708807	50		462463		3758315
79		528689	5	1199569	49		507464		4265779
78		833814		2033384	48		554435		4820215
77	5	131559		3348979	47		604845		5425059
76		205268		5401663	46		658778		6083837
75		315397		8555635	45		716867		6800704
74		474251	6	1329815	44		779630		7580334
73		696643		2026458	43		846798		8427131
72		990131		3016589	42		918880		9346011
71	6	136709		4383678	41		994302	9	1034031
70		182792		6211599	40	8	107263		1141294
69		236417		8575774	39		115303		1256598
68		300585	7	1158162	38		123670		1380267
67		378404		1536566	37		132546		1512813
66		471808		2008373	36		142062		1654876
65		581481		2589855	35		152295		1807171
64		711214		3301069	34		163396		1970566
63		864341		4165410	33		175220		2145786
62	7	103358		5198989	32		187773		2333559
61		122023		6419218	31		200833		2534392
60		142077		7839990	30		214301		2748693
59		163637		9476356	29		228179		2976872
58		187138	8	1134773	28		242784		3219656
57		212691		1347464	27		258209		3477865
56		240194		1587658	26		274444		3752309

TABLE IV. *continued.*

Yost. Age.	x.	Column A.	x.	Column B.	Yost. Age.	x.	Column A.	x.	Column B.
25	8	291584	9	4043893	10	8	686399	10	1121850
24		309973		4353866	9		726535		1194504
23		329323		4683189	8		770120		1271516
22		349730		5032920	7		817692		1353285
21		371109		5404029	6		870430		1440328
20		393639		5797668	5		928213		1533149
19		416844		6214512	4		992558		1632405
18		441268		6655780	3		107682		1740087
17		466801		7122582	2		118357		1858444
16		493555		7616137	1		132663		1991107
15		521314		8137451	0		178346		2169453
14		550566		8688017					
13		581277		9269294					
12		614038		9883333					
11		648773	10	1053211					

TABLE IV. *continued.*

For *Annuities* on Two Joint Lives.

Part 3.—*Difference of Age* 11 *Years.*

Yo^st. Age.	x.	Column A.	r.	Column B.	Yo^st. Age.	x.	Column A.	r.	Column B.
85	2	189000	2	1890000	55	7	246794	8	1648883
84		507520		6965200	54		277436		1926319
83	3	167107	3	2367592	53		310802		2237121
82		475142		7119017	52		347167		2584289
81	4	114974	4	1861639	51		386443		2970732
80		224034		4101984	50		428285		3399017
79		385366		7955640	49		471842		3870859
78		606197	5	1401761	48		516905		4387764
77		951648		2353409	47		564423		4952187
76	5	149401		3847417	46		615844		5568031
75		231060		6158019	45		670862		6238893
74		353222		9690235	44		730120		6969013
73		528943	6	1497966	43		793782		7762795
72		766102		2264068	42		861895		8624690
71	6	107857		3342639	41		933943		9558634
70		147413		4816769	40	8	100858	9	1056721
69		195752		6774285	39		108665		1165386
68		252069		9294977	38		116741		1282128
67		318797	7	1248295	37		125193		1407321
66		399486		1647781	36		134159		1541479
65		496706		2144487	35		143799		1685278
64		610630		2755117	34		154259		1839538
63		745164		3500281	33		165477		2005014
62		903720		4404001	32		177388		2182402
61	7	107709		5481091	31		189994		2372396
60		126693		6748019	30		203142		2575538
59		146926		8217281	29		216572		2792110
58		168767		9904953	28		230571		3022681
57		192827	8	1183322	27		245257		3267938
56		218768		1402090	26		260716		3528655

TABLE IV. *continued.*

Yo^st Age.	x.	Column A.	x.	Column B.	Yo^st. Age.	x.	Column A.	x.	Column B.
25	8	276982	9	3805637	10	8	654554	10	1063632
24		294255		4099892	9		693358		1132968
23		312675		4412567	8		735121		1206480
22		332169		4744736	7		780840		1284564
21		352664		5097400	6		831241		1367688
20		374197		5471597	5		886914		1456380
19		396610		5868207	4		948434		1551223
18		419894		6288101	3	9	102881		1654105
17		444321		6732422	2		113010		1767115
16		470008		7202431	1		126614		1893729
15		496671		7699101	0		169972		2063701
14		524584		8223685					
13		554090		8777776					
12		585363		9363139					
11		618631		9981771					

ERRATA,

In the *Doctrine of Life Annuities and Assurances*, in addition to those pointed out in page 622.

Page.	*Line.*	*For*	*Read.*
5,	22,	$\frac{3}{5} \times \frac{4}{10} = 100$	$\frac{3}{5} \times \frac{4}{10} \times 100$
36,	last,	·4407142 [*in the last col.*]	·4407132
42,	13,	(1+)	$(1+\rho)$
174,	2,	1402·171	1032·097
175,	2,	1402·171 × ·05	1032·097 × ·05
——	—	70·109, or £70 : 2 : 2	51·605, or £51 : 12 : 1
179,	25,	340·656, or £340 : 13 : 1	338·581, or £338 : 11 : 7
——	26,	40·656	38·581
190,	13,	$\frac{(1-B'C)b'}{(1+\rho)}$	$\frac{(1+B'C)b'}{(1+\rho)}$
236,	2,	$(a'-a')$	$(a-a')$
289,	16,	$\frac{a.\text{HBC}}{2a}$	$\frac{a.\text{HBC}}{2a}$
423,	17,	XVI.	XVIII
428,	11,	11·035	11·038
436,	4,	XXVIII	XXVII
491,	23,	London Annuity	London Annuitant
543,	last,	5049	5047

In page 516, line 23, for "60 and 81," read "50 and 81;" Government having extended their plan, by a subsequent act, since that note was written.

In page 482, line 13, for "£180," read "£200;" the *Amicable Society* having increased the sum which they guaranty to the shareholders, since that passage was written. I would also observe, in explanation to what I have said in the first note in page 483, that the *Amicable Society* have *since* published a table of the *rates* by which they effect assurances on single lives. These rates are much lower (particularly when the assurance is for a *term of years* only) than those of any other office: and it is the *first* instance of any *reduced rates* being offered to the public.

In this *Appendix*, page 9, line 2 from the bottom, for *London Life Assurance*, read *London Life Association*.